GPS-UTM Module 7:
*What Does a Surveyor Do?*

**Topics Covered:** surveying, trigonometric applications, level curves, functions of two variables

**Required Background Material:** GPS-UTM Module 6

**Introduction**

A *surveyor* is someone who measures and draws what the earth’s surface looks like. This information is needed for deeds, leases, and other legal documents. One of the most famous surveyors was George Washington, who did much of the original surveying along the Kanawha River and around Point Pleasant.

The basic ideas behind the science of surveying come from trigonometry, as we partially showed in Module 6; so the drawings of polygonal regions in the figure at right should look familiar to you. The overall illustration [1] shows the tools of the surveying trade in 1728. Most of these devices were used to measure either distance or angles.
In the 20th century, the primary surveying tools were linear measuring devices, barometers (to measure elevation), and transits (to measure angles). The most common 20th century surveying device, the transit, is still in use. (See photos below.) Today’s surveyors use GPS instruments, calculators, computers, and high tech barometers. A device, such as the modern one shown on the right, may contain all of those instruments as well as a transit.

Using a Transit Instrument

A Modern Transit

Surveyors use trigonometric ratios to indirectly measure distances that they can’t measure directly. Suppose, for instance, you have the situation shown below.
If you would like to measure the length $AB$, as shown in the diagram; but if you can’t walk on water, you can instead measure the length $AC$ and then measure angle $A$ using the transit or a simple protractor. You know that

$$\cos A = \frac{AC}{AB}.$$ 

So, if $AC = 100 \text{ meters}$ and $m\angle BAC = 30^\circ$, $\cos 30^\circ = \frac{100}{AB}$.

But, when you look up $\cos 30^\circ$ in a trig table or calculator, you will find out that it is equal to $\frac{\sqrt{3}}{2}$.

Thus,

$$\frac{\sqrt{3}}{2} = \frac{100}{AB}$$

$$\sqrt{3}AB = 200$$

$$AB = \frac{200}{\sqrt{3}} \approx 115.5 \text{ meters}.$$ 

**Problem 1**

From a height of 2 yards off the ground, a person measures the angle of elevation to the top of a tower which is 50 yards away. The angle measures $64^\circ$. How tall is the tower? Round to the nearest yard.
Problem 2

Find a river, ravine, or lake that is too large to cross and use the method of Problem 1, or the example on p. 2, to determine its width. If you set up a right triangle, you will only need use your GPS to find the length of one side. (i.e. You will only need to take two GPS readings.) A protractor can be used to measure one of the acute angles.

Turn in a drawing of the physical situation and the triangle you are using. Also, describe your method of solution and show your calculations.

Problem 3

As we saw in Module 6, a surveyor doesn’t need to have a right triangle in order to determine areas and distances. We illustrate this with the following problem.

a) To determine the distance $AB$ across a steep canyon, Kelly walks 600 yd from $B$ to another point $C$. She then finds that $m \angle ACB = 35^\circ$ and $m \angle CBA = 106^\circ$. Find $AB$. 

![Diagram of a canyon with points A, B, and C labeled.](image-url)
b) Compare this to Problem 2. What are the advantages and/or disadvantages of using a right triangle, as compared to an arbitrary triangle?

**Isoclines**

A surveyor is not only interested in latitude and longitude (or Easting and Northing), but he or she also records elevation. If you have a fairly new GPS device, you may have noticed that it gave the elevation above sea level for every point at which you checked the coordinates.

A line on a map that contains only points that have the same elevation is called an *isocline*. These can be very useful when you need to use a two-dimensional graphic to describe terrain.

**Problem 4**

Consider the picture on the next page. It shows the isoclines on the snowcap of Mt. Rainier in Washington State, enhanced by color and shading. This is an example of what is called a *topographical* map.

![Topographical Map of Mt. Rainier](image)
a) Highlight in red where you think the steepest places on the mountain are.

b) Highlight in green where you think the mountain is the flattest.

c) What is the difference in elevation between the dark blue lines?

d) What is the difference in elevation between the blue lines?

**Problem 5**

This problem is for your entire class to work on together. Go to a large hilly region near your school and take as many GPS readings as you can. Include the Easting, Northing, and elevation of as many points as possible. Take the readings further apart in flatter areas and closer together in steeper areas. Try to cover as much of the area as possible.

When you get back to class, find out who has the largest and smallest Northing numbers, and who has the largest and smallest Easting numbers. Draw lines at the top, bottom, and sides of a large piece of chart paper (as shown below) and label the chart with those numbers.
Make a grid on this paper, and record what Easting or Northing number each grid line represents. (This is similar to how the map of West Virginia is laid out below.)

Example of a Grid Marking Easting and Northing Numbers.

Then, on a separate sheet of paper, make a list of all the elevation numbers you have collected and assign a colored pencil or crayon to each number. (It will look especially nice if you use darker bluer colors for low elevations and bright reds and yellows for high elevations.) If you have a too many numbers, you may need to bin the numbers and assign a color to each bin. Each student should then plot the points he measured, using the appropriate colored pencil. Finally, connect the dots of the same color that are near to each other. These connecting lines will be isoclines. Finish the chart by making a colored “key” which tells the elevation of each isocline.
**Problem 6**

What do you think an *isotherm* is?

**Problem 7 (Opt.)**

Isoclines and isotherms are examples of *level curves*, curves that show where a particular measure has a constant value. It is easier to draw level curves if you know the relationship between two variables, most commonly $x$ and $y$. For example, consider the relationship $xy$. We will refer to this relationship by saying that we have a *function of x and y*. The notation is $f(x, y) = xy$.

To find the level curves, we assign various values to $f(x, y)$. First let $f(x, y) = xy = 1$. On a piece of paper, plot various ordered pairs $(x, y)$ that satisfy this relationship. Then join the points in the same manner as you did in Problem 5. Or solve for $y = \frac{1}{x}$, use a calculator to see the graph, and then transfer the graph to your paper.

Repeat this process with $f(x, y) = xy = k$ for $k = 2, 3, 4, \frac{1}{2}, -1, -2, -3, -\frac{1}{2}$. Graph all of the level curves on the same paper, using a different color for each value of $k$.

**References**