GPS-UTM Module 2: How Far Away Do I Live?

**Topics Covered:** Pythagorean Theorem, Distance formula, Unit conversion

**Required Background Material:** GPS-UTM Module 1, familiarity with right triangles, some experience with the Pythagorean Theorem

**Introduction**

The *Pythagorean Theorem* is the most famous theorem of the ancient Greeks. It states that the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two legs of a right triangle. (See diagram and illustration below.)

We can use this theorem to find the distance between two points. As shown in the illustration on the right, the distance from the point (3,4) to the origin (0,0) can be determined by drawing a right triangle whose legs have length 3 and length 4. Then the distance is $c = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$. 

![Pythagorean Theorem Diagram](image)
If, instead of the points (3,4) and (0,0), we use \((x_1, y_1)\) and \((x_2, y_2)\), the lengths of the two legs become \(|x_2 - y_2|\) and \(|y_2 - y_1|\). This gives the well known distance formula for the distance between two arbitrary points.

\[
d = \sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}
\]

**Problem 1**

Here are the UTM coordinates of two points in Mason County. Locate and mark the points on the Mason County grid which is shown on the next page.

Point A: 17 S 4300000  
Point B: 17 S 4280000

Using the distance formula, find the distance between Point A and Point B. [Hint: Use \((x_1, y_1) = (400000, 4280000)\) and \((x_2, y_2) = (420000, 4300000)\).] Write this number in the space provided below.

\[
d(A, B) = \text{__________________________ meters}
\]
Does it make any difference which point you call \((x_1, y_1)\) and which point you call \((x_2, y_2)\)? Why or why not?

All of the numbers in a UTM system represents meters, and that is not how distances in Mason County are usually measured. In order to change the distance to miles, we will need to use a conversion factor. You have probably used conversion factors before. For instance, you probably know that

1 Tablespoon = 3 teaspoons
That means that $1 = \frac{3 \text{ teaspoons}}{1 \text{ Tablespoon}}$. This is an example of a conversion factor. You can multiply by a conversion factor and it will not change the amount you started with; it will just write it in a different way. To change 6 teaspoons into tablespoons, we would do this:

$$6 \text{ teaspoons} = 6 \text{ teaspoons} \times \left( \frac{1 \text{ Tablespoon}}{3 \text{ teaspoons}} \right) = 2 \text{ Tablespoons}$$

It was easy to figure out which way to turn the conversion factor. We turned it over so that the teaspoons cancelled, a technique called *dimension analysis*.

**Problem 2**

Draw a line between Point A and Point B on the map given on p.2. Convert $d(A, B)$ into miles, and write the number in the space below. [Hint: There are 5,280 feet in a mile, and there are 3.28 feet in a meter.]

$$d(A, B) = \text{______________ miles}$$

**Problem 3**

Using your GPS, find the UTM location of your school and your home. Write these on the spaces provided below.

**SCHOOL**

Northing _________________       Easting _________________

**HOME**

Northing _________________       Easting _________________

Using the letters S and H, plot these locations on the map of Mason County shown on p.5. Then find the distance from your home to your school. Record the distance in the chart below, using both meters, miles, and kilometers. [Hint: A kilometer is 1000 meters.]

<table>
<thead>
<tr>
<th>Distance from Home to School</th>
<th>Meters</th>
<th>Miles</th>
<th>Kilometers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Problem 4 (Opt.)**

Find the driving distance from your home to your school, and fill out the following chart.

<table>
<thead>
<tr>
<th>Driving Distance from Home to School</th>
<th>Meters</th>
<th>Miles</th>
<th>Kilometers</th>
</tr>
</thead>
</table>

Compare these measures to the distances you recorded in Problem 3. Why are they different?