# FENCING BY RIVER OPTIMIZATION

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Fencing Optimization Abstract

We will be using the calculus method of optimization in order to determine the dimensions of the largest enclosure made from 100 yards of fencing that is adjacent to a river. After doing so, we will do it again in a case where the river has a bend in it. We will begin by creating a general model of the situation to illustrate the general case. Then we will determine the area formula which we are trying to maximize, as well as the constraint formula for how much fencing is available to work with. Next, we will isolate the y variable in the constraint formula, and substitute into the area formula. After the substitution we will differentiate that equation, then set it to 0 and find the critical points. The critical point we find should be the maximum value for x. Finally, once we have that number we will use the first derivative test to justify it being a maximum and then plug it back into our previous equation to solve for the area.

Fencing Optimization

We know that there is a maximized area for a rectangular fence with a fixed perimeter

This was found by:

$$Area = length * width$$

And
$$Perimeter = 2 * length + 2 * width$$

If the perimeter is set to 100 Yards, then the equation for perimeter would be:

$$100 = 2 * length + 2 * width$$

Now if one side Is a straight edge that does not need fencing, then the equation would be

$$100 = length + 2 * width$$

Rearranging the equation leads to:

$$length = 100 - 2 * width$$

This can then be substituted into the area equation to make:

$$Area = (100 - 2 * w) * w$$

Now to optimize, the derivative was taken:

$$A' = 100 - 4w$$

When set to zero, the critical point of 25 was found. This value was then plugged into the substituted area formula and found that the max area of a perimeter of 100 yards was 1250 square yards.

The same style of optimization was used for a rectangular area with a straight river on 2 sides, making the new perimeter equation:

$$length = 100 - width$$

The new area equation was found to be:

$$Area = (100 - w) * w$$

And its derivative:

$$A' = 100 - 2w$$

When set equal to zero, the critical point of 50 was found. This was plugged into the substituted Area equation and found that the max area was 2500 square yards.

Now the question is, what if the river is curved inward, what would the maximum possible are be, and where?

To do this, the curve of the river was modeled by the equation $x^2$. Thus making the area equation:

$$\int_{x\ initial}^{x\ final} x^2 dx$$

The perimeter of the new style of fencing was broken into 2 vertical legs, A and C, and 1 horizontal leg, b. These values were set to be equal to 100:

$$A + B + C = 100$$

It was then decided that:

$$A = Area(x_{initial})$$

$$C = Area(x_{final})$$

$$B = x_{final} - x_{initial}$$

It was found that the variables were dependent on one another, so the symmetry across the y axis of the equation of $x^2$ was used. It was decided, that if there were negative values of x, then the absolute value could be used to determine the distance between $x_{initial}$, and $x_{final}$:

$$\Delta x = x_{final} - x_{initial}$$

After several tries, a single variable substitution was not found, so tables were made to show the trend of the Area for different scenarios: Leg A<Leg B, Leg A=Leg B, and Leg A=0. These values were chosen due to the symmetry across the y-axis, if input x values were multiplied by -1, then the same results would occur.

It was decided that the if one leg was chosen to be a value, and added to the absolute value of $x_{initial}$ from 0, then there would be a different value at some point that would add to Area($x_{final}$) to give an addition of 100:

$$100 = x_{final} + x_{final}^2 + x_{initial} + x_{initial}^2$$

These values were then found in the table along with the areas. Also, the area values were then compared with the other leg scenarios.

## Conclusion

It was found that the greatest area occurred when one of the vertical legs became equal to 0, giving an area between 2903.2, and 2995.36 square yards. With more time and a better number processor, the exact maximum area could be found, along with the optimized equation. Overall, the trend is as leg A or C go to 0, then the maximum area is achieved which is between 2903.2 square yards, and 2995.36 square yards, trending towards approximately 2950 square yards.