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MTH 230

27 March, 2018

Project: Does it Really Make Sense to Aim for the Center of Mass

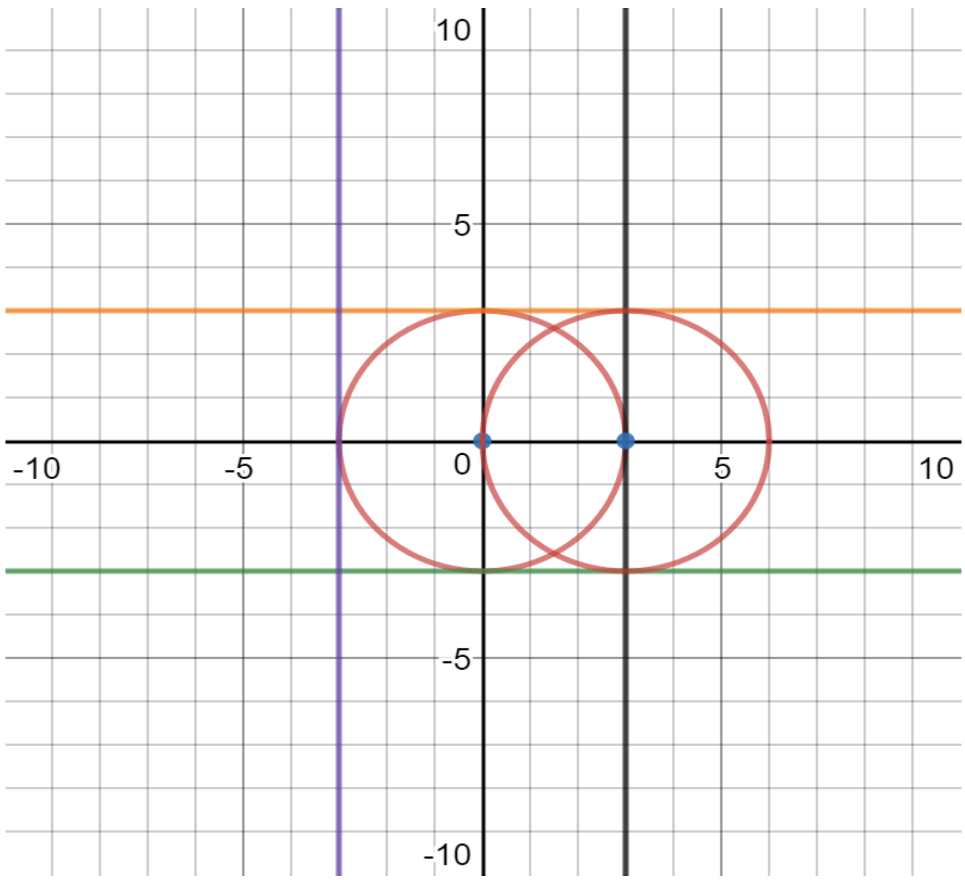
Abstract:

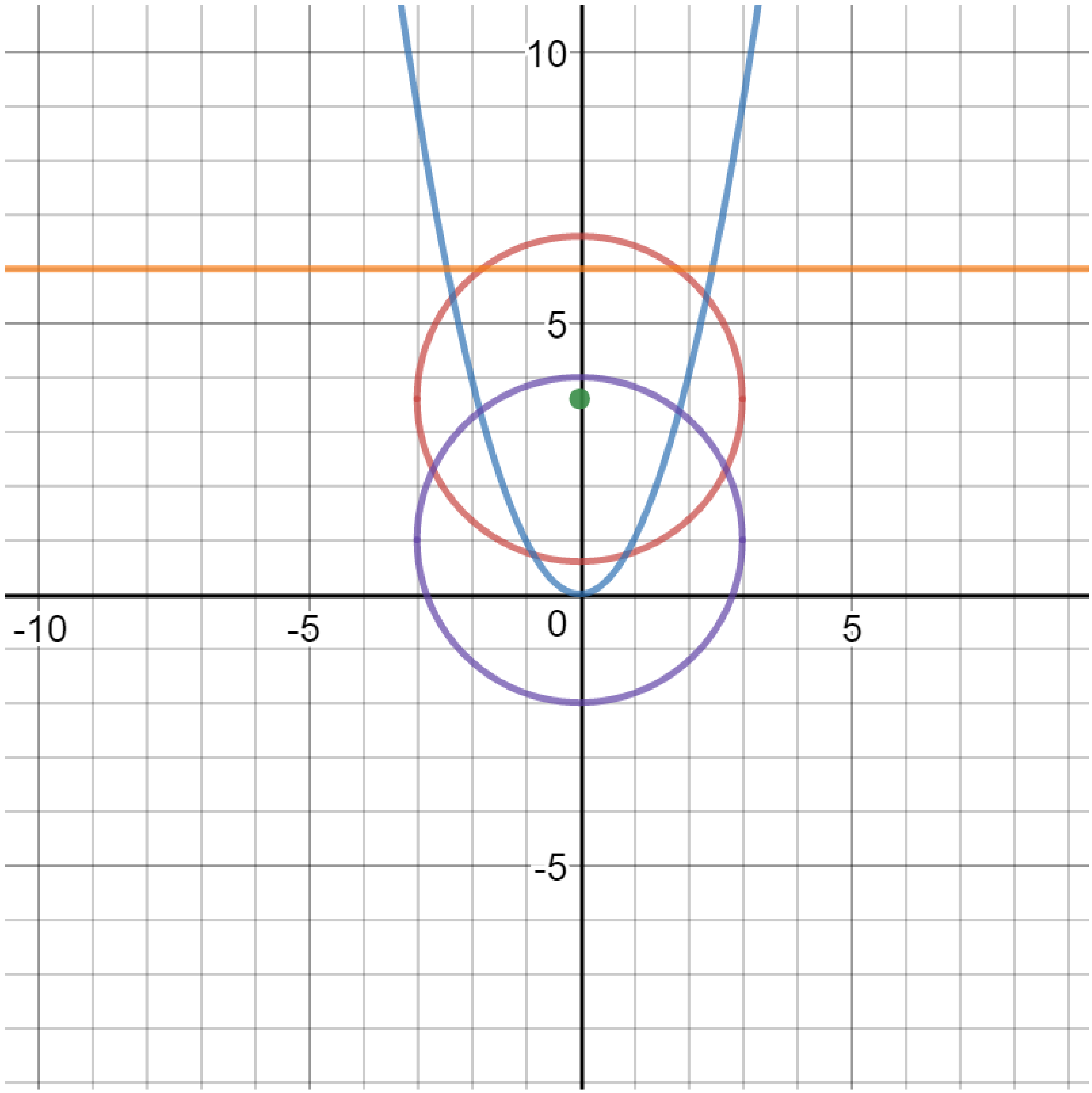
In this project, we will use calculus to find the center of mass of symmetrical and nonsymmetrical objects to determine if aiming for the center of mass would make sense. To do this, we will show three separate examples of targets and their differing centers of mass. The first object will be a square with side lengths equaling 6 inches. The second object will be the area between the graph of y= 6 and the parabola of y= x^2. The final object will show a rectangle with a height of six inches and a length of nine inches that is denser on a certain side than the other. In this scenario, a sniper will be aiming at the targets described above. The shooter will have a probable hitting area that is circular with a three-inch radius. After finding the center of mass of each object by using the center of mass formula, we will set that point as the aiming point of the shooter. We will then determine if it makes sense to aim for the center of mass by visual comparison of the targets with differing aiming points.

Procedure:

We first graphed out the three targets that our shooter will be aiming at using DESMOS graphing calculator. After graphing these objects, we used the formula to find the center of mass and graphed that point as well. We then set the point that the center of mass was found to be at as the aiming point for the shooter and used it as the center of the probable hitting circle of the shooter. We then also graphed other plausible aiming points of the shooter that were not the same as the center of mass to provide a visual aid as to whether the aiming point made sense or not.

Visuals:




10
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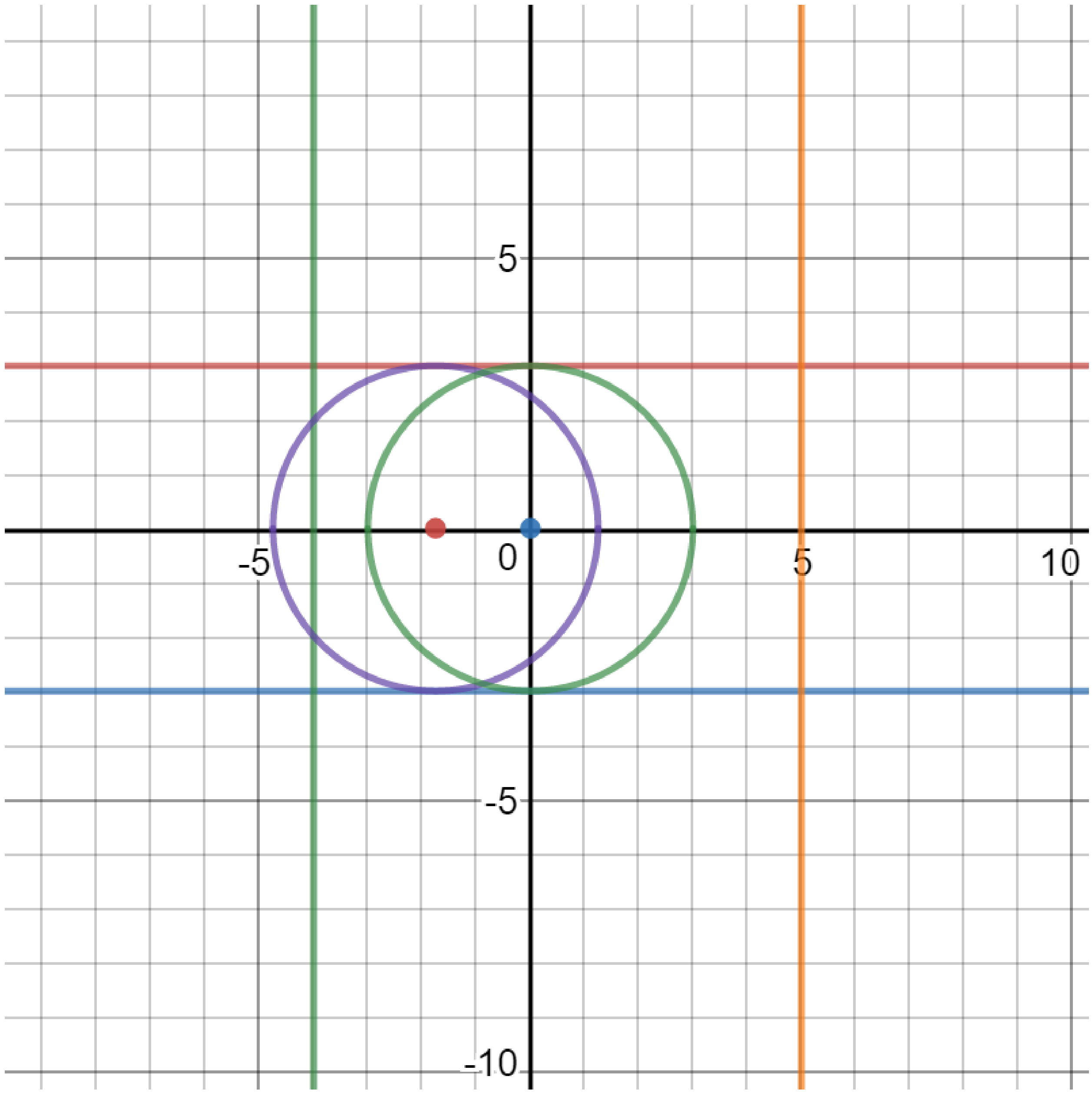
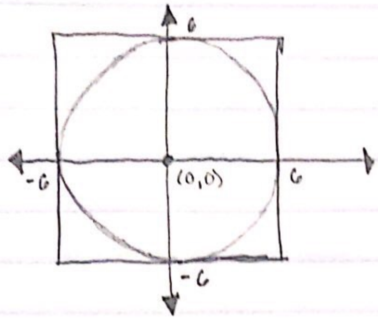


Finding the Center of Mass:

In order to find the center of mass, we had to use skills that we have learned in Calculus 2. For some of our objects, the center of mass was easy to find due to the line of symmetry principle which states that if an object is symmetrical about an axis then the center of mass will lie on that axis. However, to show our competence, the math will be shown for all targets.

Square:



Center of Mass for Square: Let $f(x) = 6$, $g(x) = -6$ on $(-6, 6)$

A = Area of the Square = $(L)(W) = (6)(6) = 36 \text{ in}^2$

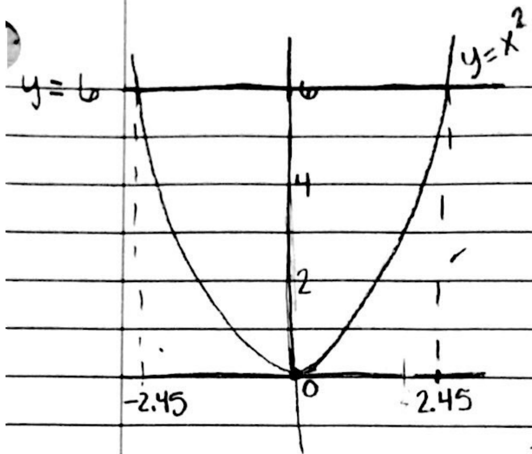
$$\bar{x} = \frac{1}{A}\int_a^b x\,[f(x) - g(x)]\,dx$$
$$= \frac{1}{36}\int_{-6}^{6} x\,[6 - (-6)]\,dx$$
$$= \frac{1}{36}\int_{-6}^{6} 12x\,dx$$

Fundamental Theorum of Calculus

$$= \frac{1}{36}\left(6x^2\right)\Big|_{-6}^{6}$$
$$= \frac{1}{36}\left[(6(6)^2) - (6(-6)^2)\right]$$
$$= \frac{1}{36}\left[(216) - (216)\right]$$
$$= \frac{1}{36}(0) = \boxed{0}$$

$$\bar{y} = \frac{1}{A}\int_a^b \frac{1}{2}\left([f(x)]^2 - [g(x)]^2\right)dx$$
$$= \frac{1}{36}\int_{-6}^{6} \frac{1}{2}\left((6)^2 - (-6)^2\right)dx$$
$$= \frac{1}{36}\int_{-6}^{6} \frac{1}{2}(36 - 36)\,dx$$
$$= \frac{1}{36}\int_{-6}^{6} \frac{1}{2}(0)\,dx$$
$$= \frac{1}{36}\int_{-6}^{6} 0\,dx$$
$$= \frac{1}{36}(0) = \boxed{0}$$

Center of Mass = (0,0)



$$\sqrt{x^2} = \sqrt{6} \qquad x = \pm 2.45$$

$$\int_{-2.45}^{2.45} (6 - x^2)\,dx = \left[6x - \frac{x^3}{3}\right]\Big|_{-2.45}^{2.45}$$

$$= \left[6(2.45) - \frac{(2.45)^3}{3}\right] - \left[6(-2.45) - \frac{(-2.45)^3}{3}\right]$$

$$= [14.7 - 4.90204] - [-17.7 + 4.90204]$$

$$A = 19.5959$$

$$\bar{x} = \frac{1}{A}\int_{-2.45}^{2.45} (6 - x^2)\,dx = \frac{1}{19.5959}\int_{-2.45}^{2.45} x(6 - x^2)\,dx$$

$$= \frac{1}{19.5959}\int_{-2.45}^{2.45} (6x - x^3)\,dx = \frac{1}{19.5959}\left(\frac{6x^2}{2} - \frac{x^4}{4}\right)\Big|_{-2.45}^{2.45}$$

$$= \frac{1}{19.5959}(8.999 - 8.999) = \frac{1}{19.5959}(0) \qquad \bar{x} = 0$$

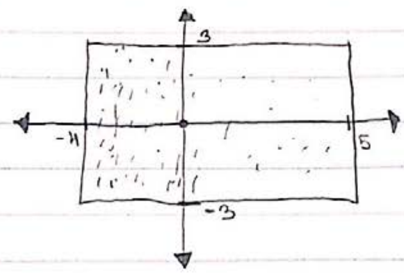
$$\bar{y} = \frac{1}{A}\int_{-2.45}^{2.45} 1/2\left(6^2 - (x^2)^2\right)dx = \frac{1}{19.5959}\int_{-2.45}^{2.45} 1/2\left[36 - x^4\right]dx$$

$$\frac{1}{19.5959}\left(\frac{1}{2}\right)\int_{-2.45}^{2.45} (36 - x^4)\,dx = (0.02551554)\left[36x - \frac{x^5}{5}\right]\Big|_{-2.45}^{2.45}$$

$$= (0.02551554)[70.55 + 70.55] \qquad \bar{y} = 3.6$$

Center of mass (0, 3.6)

Rectangle with a more dense side:



Center of Mass:

$$c = \frac{\Sigma \text{ total moment}}{\Sigma \text{ mass}}$$

$$\begin{aligned} \text{total moment} &= 50(-4) + 40(-3) + 30(-2) + 20(-1) + 10(0) + 10(1) \\ &\quad + 10(2) + 10(3) + 10(4) + 10(5) \\ &= -350 \end{aligned}$$

$$\begin{aligned} \text{total mass} &= 50 + 40 + 30 + 20 + 10 + 10 + 10 + 10 + 10 + 10 \\ &= 200 \end{aligned}$$

$$\begin{aligned} c &= \frac{-350}{200} \\ &= -1.75 \end{aligned}$$

By line of symmetry principle,
$y = 0$

Center of Mass = (-1.75, 0)

Conclusions:

In order to determine if aiming for the center of mass made sense, we compared the probable aiming circle centered at the center of mass to other possible aiming points on the target. After examining each target, we have reached different conclusions depending on the type of target. For targets that are symmetrical about both axes, like the square, it makes sense to aim for the center of mass. The center of mass is the optimal position for aiming since all of the probable hitting circle is within the target. For targets such as the parabola, it makes sense to aim for the center of mass as well. Although not all of the probable hitting circle was within the target, the hitting circle covered a great majority of the target. This aiming point is far better than other possible aiming points like the one shown on graph. Finally, for targets with areas that have differing densities it depends on the type and shape of the target. For our example, it does not make sense to aim for the center of mass. A better aiming point would be the very center of the target since a greater portion of the probable hitting circle will be over the target.