

The proper way to throw a ball off a 100 story building

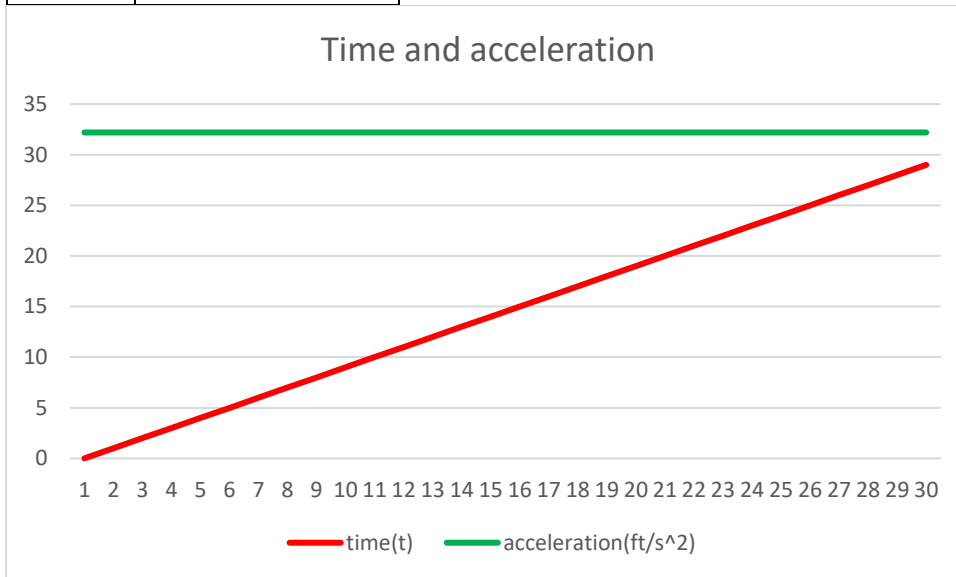
Professor Saveliev asked me to find a way to throw a ball off a 100 story building so that it would hit the ground at exactly 100 feet per second. The objective of this project is to find out if you were to throw a ball off a 100 story building at a certain rate so that it hits 100 feet per second. It is my goal to see if it is possible or not.

The first thing I did was find out the average height of one story on a building would be. According to a forum called Skyscraper Page, a commercial building one story can range from 13ft to 15ft (Pingyao, 2009). The best way to figure out the height of a classroom, like Professor Saveliev class for instance. Look at the concrete blocks that are the walls, each one is roughly one foot. If you counted them and the two that are cut at the top and bottom of the wall the classroom is 13.5ft tall give or take a couple of inches. For my experiment I am going to use the highest of a story which is 15 feet tall. I decided to go with 15 feet because the classmate, Tyler Zubick, before me used 13 feet and I did not want to have to similar of outcomes as he had (Zubick, 2016).

I now have the height that I want to use for this experiment, I need to find a equation that would best fit the problem I am trying to solve. Looking at this problem the same way as Tyler did I am going to take the acceleration equation which is $(dy)'=a$ (Zubick, 2016). Taking the antiderivative of that equation which gives us $(dy) = at$. Then again taking it one step further, took the antiderivative of that leaving us with $dy = \frac{1}{2} * at^2$. The following is the chart along with the graph that shows the "Time vs Acceleration". As well as the chart(s) and graph(s) for "Time vs Velocity" and "Time vs Height".

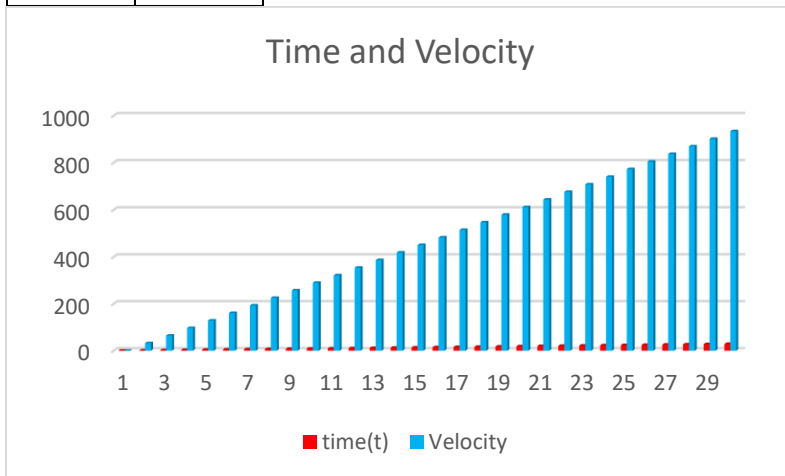
time(t)	acceleration(ft/s ²)
0	32.2
1	32.2
2	32.2
3	32.2
4	32.2
5	32.2
6	32.2
7	32.2
8	32.2
9	32.2
10	32.2
11	32.2
12	32.2
13	32.2
14	32.2
15	32.2
16	32.2
17	32.2

18	32.2
19	32.2
20	32.2
21	32.2
22	32.2
23	32.2
24	32.2
25	32.2
26	32.2
27	32.2
28	32.2
29	32.2



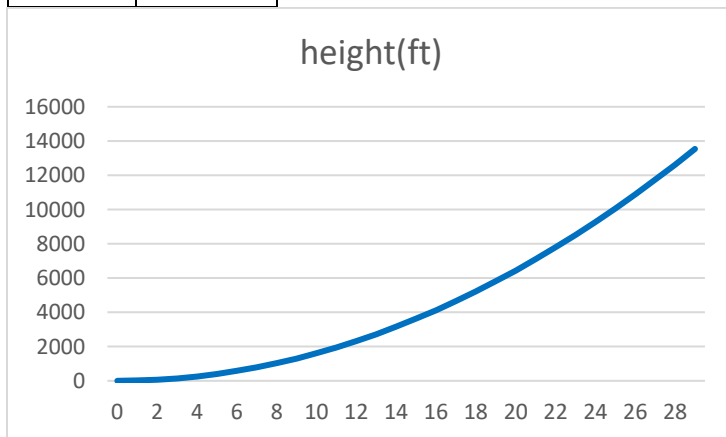
time(t)	Velocity
0	0
1	32.2
2	64.4
3	96.6
4	128.8
5	161
6	193.2
7	225.4
8	257.6
9	289.8
10	322
11	354.2
12	386.4

13	418.6
14	450.8
15	483
16	515.2
17	547.4
18	579.6
19	611.8
20	644
21	676.2
22	708.4
23	740.6
24	772.8
25	805
26	837.2
27	869.4
28	901.6
29	933.8



time(t)	height(ft)
0	0
1	16.1
2	64.4
3	144.9
4	257.6
5	402.5
6	579.6
7	788.9
8	1030.4
9	1304.1

10	1610
11	1948.1
12	2318.4
13	2720.9
14	3155.6
15	3622.5
16	4121.6
17	4652.9
18	5216.4
19	5812.1
20	6440
21	7100.1
22	7792.4
23	8516.9
24	9273.6
25	10062.5
26	10883.6
27	11736.9
28	12622.4
29	13540.1



After getting all the information I needed to finish the experiment. I then started to use the formula $dy = \frac{1}{2} * a * t^2$. In the formula “ dy ” is the height of the story, “ a ” is the 32.2 feet per second(acceleration) due to the gravity pull which is a given (Helmets, 2016), and “ t ” is the time time. Below is the equation worked out to get an estimated time on how long the ball will take to hit the ground.

$$dy = \frac{1}{2} * a * t^2$$

$$1500 = \frac{1}{2} * (32.2 \text{ feet/sec}) * t^2$$

$$1500 = 16.1 * t^2$$

$$t^2 = 93.17(\text{feet}/\text{sec})$$

$$t = \sqrt{93.17 \text{feet}/\text{sec}} = 9.65 \text{ sec}^2$$

According to the math I did above it will take the ball approximately 10 sec to hit the ground. Taking this information above to find the velocity for the ball dropping off a 100 story building so that it will finish at 100 feet per second. Using the equation $V_f = V_o + at$. V_f is the final velocity and V_o is the initial velocity, and at is the acceleration due to gravity and time.

$$V_f = V_o + at$$

$$100\text{ft}/\text{sec} = V_o + (32.2) \cdot (10 \text{ sec})$$

$$100\text{ft}/\text{sec} = V_1 + 322\text{ft}/\text{sec}$$

$$V_1 = -222 \text{ ft}/\text{sec}$$

The answer to the equation above tells me that for a ball to be dropped straight down I would have an initial velocity of 222ft/sec which is unrealistic.

time(t)	Velocity	initail velocity to get 100ft p sec
0	0	0
1	32.2	67.8
2	64.4	35.6
3	96.6	3.4
4	128.8	-28.8
5	161	-61
6	193.2	-93.2
7	225.4	-125.4
8	257.6	-157.6
9	289.8	-189.8
10	322	-222
11	354.2	-254.2
12	386.4	-286.4
13	418.6	-318.6
14	450.8	-350.8
15	483	-383
16	515.2	-415.2
17	547.4	-447.4
18	579.6	-479.6
19	611.8	-511.8

20	644	-544
21	676.2	-576.2
22	708.4	-608.4
23	740.6	-640.6
24	772.8	-672.8
25	805	-705
26	837.2	-737.2
27	869.4	-769.4
28	901.6	-801.6
29	933.8	-833.8

The chart above comes from excel where I used my findings of time which was 10 sec, followed by the velocity. The gravity of each second to that 10 seconds it needs -222 ft/sec to reach the goal of 100ft/sec at impact.

Looking at the excel spread sheet and chart, it occurred to me that I would need to throw the ball up first then letting it fall to the ground. I came to this conclusion due to the negative initial velocity.

Using the equation $V_f = \sqrt{(2 \cdot 32.2 \cdot dy) + V_i^2}$ to show the ball going from the ground up.

Vi	Height(dy)	Vf
0	1500	310.8054054
0	1450	305.581413
0	1400	300.2665483
0	1350	294.855897
0	1300	289.3440858
0	1250	283.7252192
0	1200	277.9928057
0	1150	272.13967
0	1100	266.1578479
0	1050	260.0384587
0	1000	253.7715508
0	950	247.3459116
0	900	240.7488318
0	850	233.9658095
0	800	226.9801753
0	750	219.7726098
0	700	212.3205124
0	650	204.5971652
0	600	196.570598

0	550	188.2020191
0	500	179.4435844
0	450	170.2351315
0	400	160.4992212
0	350	150.1332741
0	300	138.9964028
0	250	126.8857754
0	200	113.4900877
0	150	98.285299
0	100	80.24961059
0	50	56.74504384

The results I got from the chart above was not what I was looking for. I wanted the final velocity to decrease but instead it was increasing.

Placing all of my information into excel the results I got were still unrealistic to me. There is not a specific way to drop a ball off the top of a building. Using the same equation I used but this time I decreased the height instead of increasing it. $V_f = \sqrt{(2 \cdot 32.2 \cdot dy) + V_i^2}$.

Vi	Height(dy)	Vf
0	1500	310.8054054
0	1450	305.581413
0	1400	300.2665483
0	1350	294.855897
0	1300	289.3440858
0	1250	283.7252192
0	1200	277.9928057
0	1150	272.13967
0	1100	266.1578479
0	1050	260.0384587
0	1000	253.7715508
0	950	247.3459116
0	900	240.7488318
0	850	233.9658095
0	800	226.9801753
0	750	219.7726098
0	700	212.3205124
0	650	204.5971652
0	600	196.570598
0	550	188.2020191
0	500	179.4435844

0	450	170.2351315
0	400	160.4992212
0	350	150.1332741
0	300	138.9964028
0	250	126.8857754
0	200	113.4900877
0	150	98.285299
0	100	80.24961059
0	50	56.74504384

If you look carefully at the chart you see where I highlighted the final velocity that was the closest to 100ft/sec. The height of that is 150ft. Taking this one step further and took that equation and got the results from 160ft-150ft. You will see below that the closest to 100ft/sec is 156ft.

0	150	98.2853
0	151	98.61237
0	152	98.93836
0	153	99.26329
0	154	99.58715
0	155	99.90996
0	156	100.2317
0	157	100.5525
0	158	100.8722
0	159	101.1909
0	160	101.5086

As you can see that 156ft is the closest height where the ball will hit 100ft/sec if the initial velocity was at 0. However the finding is not what I am looking for. I want the ball to hit 100ft/sec when it impacts the ground, the chart is showing that it will hit the goal of 100ft/sec at 156ft in the air. Which is not the ground.

I have not decided to change the initial velocity of the ball. Before I had the scenario of the ball just dropping from my hand. Now I want to see the results of the ball if I were to throw the ball into the air then letting it fall 100 story's down to see if that will get me to the goal of 100ft/sec at impact to the ground. The way I found best to find the most accurate results were to change the initial velocity and see the outcome in excel. The following charts are what I got from changing the initial velocity.

Vi	Height(dy)	Vf
23	1500	311.6553
23	1450	306.4458

23	1400	301.1461
23	1350	295.7516
23	1300	290.2568
23	1250	284.6559
23	1200	278.9426
23	1150	273.1099
23	1100	267.1498
23	1050	261.0536
23	1000	254.8117
23	950	248.413
23	900	241.845
23	850	235.0936
23	800	228.1425
23	750	220.9728
23	700	213.5626
23	650	205.8859
23	600	197.9116
23	550	189.6022
23	500	180.9116
23	450	171.7818
23	400	162.1388
23	350	151.8848
23	300	140.8865
23	250	128.9535
23	200	115.7972
23	150	100.9406
23	100	83.48054
23	50	61.22908
23	0	23

Vi	Height(dy)	Vf
18.46021	1500	311.3531425
18.46021	1450	306.138497
18.46021	1400	300.8334745
18.46021	1350	295.4332062
18.46021	1300	289.9323703
18.46021	1250	284.3251297

18.46021	1200	278.6050598
18.46021	1150	272.7650626
18.46021	1100	266.7972626
18.46021	1050	260.6928832
18.46021	1000	254.4420943
18.46021	950	248.033827
18.46021	900	241.4555432
18.46021	850	234.692947
18.46021	800	227.729619
18.46021	750	220.5465469
18.46021	700	213.1215131
18.46021	650	205.4282827
18.46021	600	197.4355068
18.46021	550	189.1052071
18.46021	500	180.3906299
18.46021	450	171.2331141
18.46021	400	161.5573562
18.46021	350	151.2639394
18.46021	300	140.2169011
18.46021	250	128.2216025
18.46021	200	114.9816479
18.46021	150	100.0038967
18.46021	100	82.34548775
18.46021	50	59.6722662
18.46021	0	18.46021

Vi	Height(dy)	Vf
14.2	1500	311.1296
14.2	1450	305.9112
14.2	1400	300.6021
14.2	1350	295.1976
14.2	1300	289.6923
14.2	1250	284.0803
14.2	1200	278.3552
14.2	1150	272.5099
14.2	1100	266.5364
14.2	1050	260.4259
14.2	1000	254.1685
14.2	950	247.7532
14.2	900	241.1672

14.2	850	234.3963
14.2	800	227.4239
14.2	750	220.2309
14.2	700	212.7948
14.2	650	205.0893
14.2	600	197.0828
14.2	550	188.737
14.2	500	180.0046
14.2	450	170.8263
14.2	400	161.1262
14.2	350	150.8033
14.2	300	139.7199
14.2	250	127.6779
14.2	200	114.375
14.2	150	99.30579
14.2	100	81.49626
14.2	50	58.49479
14.2	0	14.2

As you can see above that I have tested my theory of changing the initial velocity a few different times. The best result I found was if the initial velocity was at **18.46021** that the ball will hit 100 ft/sec at the height of 150ft. But just like the result of the initial velocity of 0 the ball is still getting to 100ft/sec around the 150ft mark which is still not the ground. Below I took the chart of **18.46021** velocity and found the final velocity from the height of 155ft-150ft to see if I could get any closer to my goal.

18.46021	150	100.0039
18.46021	151	100.3254
18.46021	152	100.6458
18.46021	153	100.9652
18.46021	154	101.2837
18.46021	155	101.6011

The best results I found was still 150ft where the ball was hitting **100.0039ft/sec** as the final velocity.

18.46021	150	100.0038967
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Following up on my findings of both the initial velocity being at 0 and 18.46021. I went back to the equation I started off with which is $dy = \frac{1}{2} \cdot at^2$. But instead of plugging in 100 for dy I will plug in 156ft for the initial velocity of 0 and 150ft for the velocity at 18.46021.

$$dy = \frac{1}{2} * at^2$$

$$156\text{ft} = \frac{1}{2} * 32.2\text{ft/sec} * t^2$$

$$t^2 = 9.69$$

$$\text{time} = 3.11 \text{ seconds}$$

$$dy = \frac{1}{2} * at^2$$

$$150\text{ft} = \frac{1}{2} * 32.2 * t^2$$

$$t^2 = 9.32$$

$$\text{Time} = 3.05 \text{ seconds}$$

At the end of my experiment I have concluded that it is unrealistic to throw a ball off a 100 story building and have it reach the ground at 100 feet per second. If you wanted to get the exact results for the experiment you would have to decrease the height of which the ball is throw drastically in order to reach the proper goal.

References

Helmets. (2016, September 30). *Helmets*. Retrieved from helmets.org: <http://www.helmets.org/g.htm>

Pingyao. (2009, September 27). *Skyscraper Page*. Retrieved from <http://forum.skyscraperpage.com/showthread.php?t=173959>

Zubick, T. (2016, December 4). *Calc Project*. Retrieved from math01.com: http://users.marshall.edu/~saveliev/Teaching/Fall16/Projects/Calc_Final_Project.pdf

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Parents