Math 22 Number Theory  Spring 2000

FINAL EXAM
12 problems, 8 will be graded.

Instructions.
1. “Show enough work to justify your answers.”
2. The solutions should not make references to computations with a calculator.
3. Do the problems in the given order.
4. Start each problem on a new page.
5. Keep this sheet.

Problems.

1. Show that if \((a, b) = d\), then \((a/d, b/d) = 1\).
2. If \(a = bq + r\), then \((a, b) = (b, r)\).
3. If \((a, b) = 1\), then there are integers \(u\) and \(v\) such that \(ua + vb = 1\).
4. Find all primes less than 100.
5. The difference between two consecutive cubes is never divisible by 3 or 5.
6. State and prove the Chinese Remainder Theorem for two congruences.
7. \(x^2 \equiv 1 \mod p\) (\(p\) prime) has exactly 2 solutions \(\mod p : 1\) and \(p - 1\).
8. Prove directly the formula for \(\sigma(p^aq^l)\), where \(p, q\) are different primes.
9. Prove that \(\varphi\) is multiplicative.
10. If \(a\) and \(b\) are primitive roots of an odd prime \(p\), then \(a \equiv b^m \mod p\) for some integer \(m\). Show that \(m\) is odd.
11. Show that if a Pythagorean triple is an arithmetic progression then it has the form: \(3n, 4n, 5n, n = 1, 2, 3, \ldots\)
12. Show that \(x^{4n} + y^{4n} = z^{4n}\) has no nontrivial solutions.