Math 22   Spring 2000
EXAM 2
5 problems, 10 points each

Instructions.
1. “Show enough work to justify your answers.”
2. The solutions should not make references to computations with a calculator.
3. Do the problems in the given order.
4. Start each problem on a new page.
5. Keep this sheet.

Problems.

1. Solve the set of simultaneous congruencies: \( x^2 = 9 \mod 10, 7x = 19 \mod 24, 2x = -1 \mod 45 \). [Exercise 3.14 (d), p. 62]

2. Find the least nonnegative residue of \( 2^{68} \mod 19 \). [Example 4.1, p. 69]

3. Suppose \( G \) is a group such that the identity \((ab)^n = a^nb^n\) is satisfied for all elements \( a, b \) and 3 consecutive integers \( n \). Show that \( G \) is abelian.

4. Find a primitive root \( \mod 17 \). [Example 6.4, p.100]

5. Show that \( x = 2^{e-1} - 1, e \geq 3, \) has order 2 in \( U_{2^e} \). [Exercise 6.12, p.107]