Math 13      Spring 2000
SAMPLE PROBLEMS FOR THE FINAL EXAM

1. The graph below is the graph of \( f(x) = A \sin x + B \) for some constants \( A \) and \( B \). Determine the values of \( A \) and \( B \).

2. Is \( f(x) = 2 \sin \left( \frac{1}{2} x \right) \) a periodic function? If it is, find the period. You may graph it to decide, but then justify your conclusion symbolically.

3. The graph of function \( f \) is given below. Sketch the graph of \( f^{-1} \) (give \( x \)-, \( y \)-intercepts, etc.).

4. The height of the ball (in feet) \( t \) seconds after it is thrown is given by \( h(t) = -16t^2 + 8t + 12 \); its (upward) velocity at time \( t \) is \( v(t) = -32t + 8 \). At what time did the ball reach its maximal height? How high was it then? Explain.
5. Suppose \( f(2) = 2 \) and \( f'(x) \leq 3 \) for all \( x \). Based on the Racetrack Principle, what can you conclude about \( f(5) \) and \( f(1) \)?

6. The graph of a function \( f(x) \) is given below. Estimate the value of the derivative \( f'(x) \) of \( f(x) \) for \( x = 2, 4, \) and 7.

7. Suppose the derivative of a function \( f \) is \( f'(x) = (\sin x)^2 \). (a) On what intervals, if any, is \( f \) increasing? (b) On what intervals, if any, is \( f \) concave down?

8. Sketch the graph of the function

\[
f(x) = \frac{2x^2}{x^2 - 1}
\]

and identify its asymptotes.

9. Evaluate the following limits.

a) \( \lim_{x \to \infty} \frac{3x^3 + 1}{5x^3 + 3x + 5} \)

b) \( \lim_{x \to 2} \frac{x^2 - 4}{(x - 2)^2} \)

10. Find the equation for the line tangent to the graph of \( f(x) = \frac{1}{x^2} \) at the point where \( x = 1 \).
11. Compute the derivatives of the following functions.

   a) \(3\sqrt{x} + \sin(x^2)\)
   b) \(y^3 e^y + \arcsin y\)
   c) \(\frac{t^2}{t^3 - 1} + \ln \pi\)

12. Given \(xy + \sin y = x^3\), compute \(dy/dx\).

13. Suppose that \(F\) is an antiderivative of a differentiable function \(f\). (a) If \(F\) is concave up, what is true about \(f\)? (b) If \(G\) is another antiderivative of \(f\), what is the relation between \(F\) and \(G\)? Explain.

14. Suppose that \(F(x) = \int_{-1}^{x} f(u) du\). Express the following in terms of \(F\):

   (a) \(\int_{-1}^{3} (f(x) + 2) dx\), (b) \(\int_{-1}^{2} f(x) dx + \int_{2}^{3} f(x) dx\).

15. Let \(f\) be the function graphed below. Estimate the value of \(\int_{0}^{10} f(x) dx\) by finding the left sum and the midpoint sum each with 5 equal subintervals. (Or the same problem with the data given as a table).

![Graph of a function with data points]
16. Sketch the region between the curves $y = e^x$, $y = e$, and the $y$-axis, and find the area of the region.

17. Evaluate

$$
\int \frac{e^x}{1 + e^{2x}} dx
$$

18. Evaluate

$$
\int x \cos(1 - x^2) dx
$$

19. Decide whether the function $y(t) = 3e^{t^2/2}$ is a solution of the differential equation $y' = ty$.

20. Find the linear and quadratic approximations to $f(x) = \cos x + 1$ based at $\pi/2$.

21. Carry out two steps of Newton’s method for the equation $x^2 - 3 = 0$.

22. Find the largest possible volume of an open box made of $1 \times 1$ piece of cardboard.

23. The diameter of an air balloon is increasing at a rate of .1 foot per second. At what rate is its volume increasing when the diameter is 1 foot?


25. A problem from Section 4.11.