Math 13  Fall 1999
EXAM 4  SOLUTIONS

6 problems, 10 points each

1. Suppose \( \int_{0}^{1} f(x) \, dx = 1, \int_{0}^{3} f(x) \, dx = 1 \). Evaluate (a) \( \int_{0}^{1} 2f(x) \, dx \), (b) \( \int_{0}^{3} f(x) \, dx \), (c) \( \int_{0}^{3} f(x) \, dx \).

   Solution: (a) \( 2 \int_{0}^{1} f(x) \, dx = 2 \cdot 1 = 2 \), (b) \( \int_{0}^{3} f(x) \, dx - \int_{0}^{1} f(x) \, dx = 1 - 1 = 0 \), (c) \( \int_{0}^{3} f(x) \, dx = \int_{0}^{1} f(x) \, dx = -1 \).

2. Suppose that \( F \) is an antiderivative of a differentiable function \( f \). (a) If \( F \) is concave up, what is true about \( f \)? (b) If \( G \) is another antiderivative of \( f \), what is the relation between \( F \) and \( G \)? Explain.

   Solution: (a) Since \( F \) is concave up, \( F' = f \) is increasing. (b) Since \( F' = G' = f \), we have \( (F - G)' = f - f = 0 \), so \( F - G \) is constant.

3. Suppose that \( F(x) = \int_{-1}^{3} f(u) \, du \). Evaluate the following in terms of \( F \):

   (a) \( \int_{-1}^{3} (f(x) + 2) \, dx \), (b) \( \int_{-1}^{3} f(x) \, dx + \int_{-1}^{3} f(x) \, dx \).

   Solution: (a) \( \int_{-1}^{3} f(x) \, dx + \int_{-1}^{3} 2 \, dx = F(3) + 8 \), (b) \( \int_{-1}^{3} f(x) \, dx = F(3) \).

4. Evaluate the following: (a) \( \int_{1}^{4} \frac{1}{\sqrt{3x}} \, dx \), (b) \( \int_{0}^{2} e^{2x+1} \, dx \). (Hint: Simplify first).

   Solution: (a) \( \frac{1}{\sqrt{3}} \int_{1}^{4} x^{-1/2} \, dx = \frac{1}{\sqrt{3}} \left[ 2x^{1/2} \right]_{1}^{4} = \frac{2}{\sqrt{3}}(\sqrt{4} - \sqrt{1}) = \frac{2}{\sqrt{3}} \).
\[ (b) = e^2 \int_0^1 e^{2x} dx = e \left[ \frac{1}{2} e^{2x} \right]_0^1 = \frac{e}{2} (e^1 - e^0) = \frac{e}{2} (e - 1). \]

5. Let \( f \) be the function graphed below. Estimate the value of \( \int_0^{10} f(x) dx \) by finding the left sum and the midpoint sum each with 5 equal subintervals.

\[ Solution: \quad L_4 = 2(2+8+10+8+2) = 60, \quad M_4 = 2(5.5+9.5+9.5+5-2) = 54. \]

6. Sketch the region between the curves \( y = e^x, \quad y = e, \quad \) and the \( y \)-axis, and find the area of the region.

\[ \text{Solution:} \quad \text{The area} = \int_0^1 (e - e^x) dx = (ex - e^x)|_0^1 = (e1 - e^1) - (0 - e^0) = e - e - 0 + 1 = 1. \]