Kevin Hoops

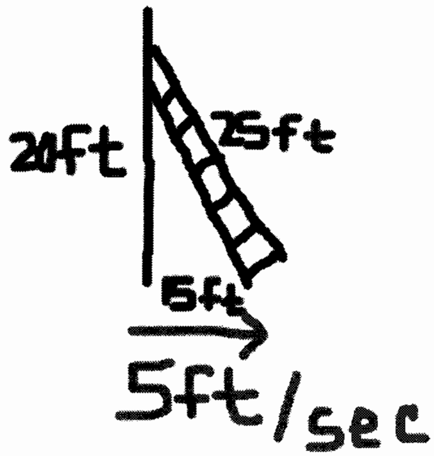
Math 231

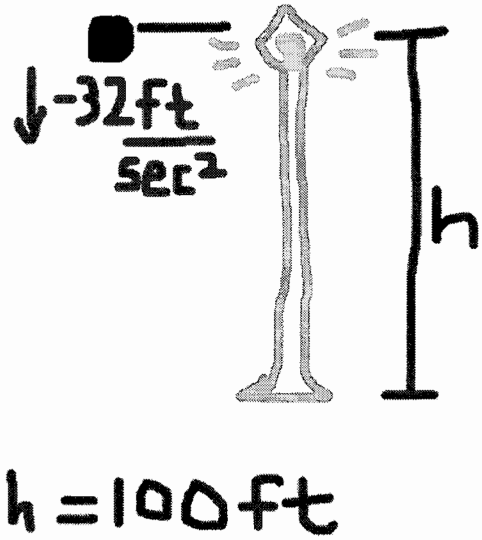
11/25/17

Project Question: How fast does the shadow of a falling ball on a sliding ladder move?

Introduction:

When faced with this question the first thing I did was draw a scenario and set up dimensions for it to be solved. The picture shows a flat landscape and a ladder against a wall, which begins to slide at a rate of 5ft/s. A ball that is 20 feet from the wall (in the horizontal direction) is 100 feet above the ground and is dropped at the same time that the ladder starts sliding. Also, there is a light post that is 100 feet tall and is 40 feet from the wall.





The Falling Ball:

The acceleration due to gravity -32ft/sec/sec and the antiderivative of this gives the velocity. Therefore, the velocity at anytime of the ball after it is released is: $v=-32t+v_0$. Since initial velocity is 0ft/s then $v=-32t$. Taking the antiderivative of this gives the position of the ball at any time and $P=-16t^2+100$. Light travels as straight lines through space which can be explained as infinite vectors that propagate off the light source. However, the light can be stopped once it meets an object that light cannot pass through. This explains why shadows are formed. Knowing this, it's easy to conclude that the angle that the ball makes with the light source at anytime is congruent to the angle that the shadow of the ball makes with the light as well. So, the angle that the ball makes with the light with ~~resect~~ respect to the horizontal is: $\tan^{-1}((100-(100-16t^2))/20)=\tan^{-1}(0.8t^2)$.

The Sliding Ladder:

The ladder in this scenario is 25 feet long and initially the ladder is against the wall 20 feet above the ground and is touching the ground 15 feet from the wall. Once the ball begins to fall the ladder starts to slide away from the wall at a rate of 5ft/sec. Therefore, the length of the ladder ~~is~~ from the wall is $(15+5t)$ for $0<=t<=2$. Since $a^2+b^2=c^2$ then the length from the ground to the top of the ladder against the wall can be found and is: $b=(625-(15+5t)^2)^{0.5}$. At any time the ladder represents a different linear equation in a 2-D plane (considering that the bottom of the wall is the origin) and can be manipulated to find the exact point on the ladder at any time that the shadow of the ball is on. $Y=mx+b$, so m would equal $-(625-(15+5t)^2)^{0.5}/(15+5t)$ and the y-intercept would be $(625-(15+5t)^2)^{0.5}$. Therefore the equation of a point on the ladder at any time is $y=-(625-(15+5t)^2)^{0.5}/(15+5t)x+(625-(15+5t)^2)^{0.5}$. The coordinates of the shadow of the ball at any time is found by: $\tan^{-1}(0.8t^2)=\tan^{-1}((100-y)/(40-x))$. This equation is then substituted for y and solved for x. The result is a parametric curve that describes the path of the shadow of the ball through space in both the horizontal and vertical directions, denoted x(t) and y(t). According to excel the shadow of the falling ball isn't on the ladder until time $t=1.69$. Therefore, the time interval for this scenario that concerns the speed of the shadow of the ball down the ladder is $[1.69,2)$.

Conclusion:

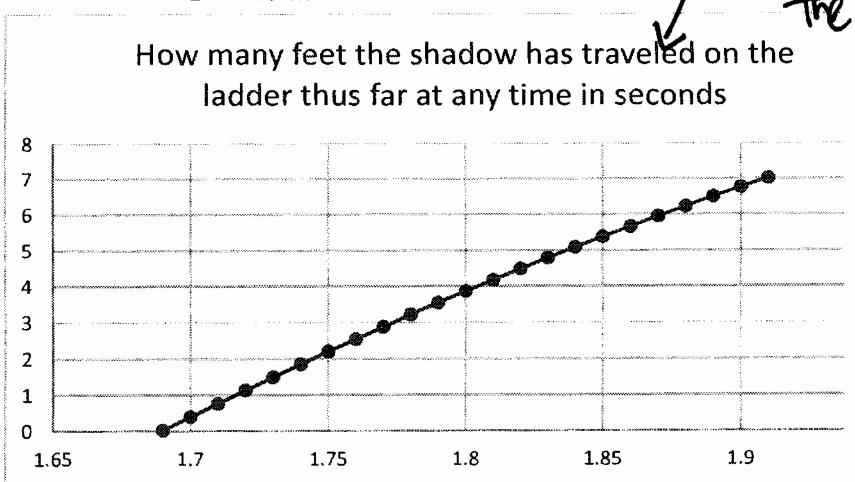
The speed of the shadow of the ball on the sliding ladder with reference to space is: $(x'(t)^2+y'(t)^2)^{0.5}$. This is the square root of sum of the squares of the derivatives of the x and y components of the parametric curve that describes the movement of the shadow of the ball through space. This is the instantaneous speed of the shadow of the ball through space at any time. However, if you're trying to find the speed of the ball with reference to the ladder then this can be solved by using the triangle similarity theorem. The ratio of the x-position of the shadow at any time to length of the ladder (in the horizontal direction) from the wall at that time is congruent to the length of the ladder already covered at a given time to the total length of the ladder. Therefore, the distance covered thus far along the ladder at any time is: (25*x(t)/(15+5t))=L(t). This results in a new equation with respect to time that was found by manipulating the x-component of the original parametric curve. Finally, the instantaneous speed of the shadow of the ball along the sliding ladder with reference to the ladder at any time is: d/dt(25*x(t)/(15+5t))=L'(t).

x-position of shadow at any time

x	f(x)
0.01	-60.3357
0.02	-60.6613
0.03	-60.9765
0.04	-61.2811
0.05	-61.5748
0.06	-61.8573
0.07	-62.1284
0.08	-62.3878
0.09	-62.6353
0.1	-62.8707
0.11	-63.0936
0.12	-63.3039
0.13	-63.5013
0.14	-63.6857
0.15	-63.8568
0.16	-64.0145
0.17	-64.1585
0.18	-64.2888
0.19	-64.405
0.2	-64.5072
0.21	-64.5951
0.22	-64.6686
0.23	-64.7276
0.24	-64.772
0.25	-64.8017
0.26	-64.8166
0.27	-64.8167
0.28	-64.8019
0.29	-64.7721
0.3	-64.7274
0.31	-64.6677
0.32	-64.593
0.33	-64.5034
0.34	-64.3988
0.35	-64.2793
0.36	-64.145
0.37	-63.9959
0.38	-63.8321
0.39	-63.6537
0.4	-63.4607
0.41	-63.2534
0.42	-63.0318
0.43	-62.7962
0.44	-62.5465
0.45	-62.2831
0.46	-62.006
0.47	-61.7156
0.48	-61.4119
0.49	-61.0951

x	f(x)	Velocity (ft/s) of Shadow of Ball with reference to the Ladder at
1.69	0.024532	
1.7	0.401423	37.68914
1.71	0.772492	37.10681
1.72	1.137788	36.52968
1.73	1.497364	35.95759
1.74	1.851268	35.39034
1.75	2.199545	34.82769
1.76	2.542238	34.26939
1.77	2.87939	33.71513
1.78	3.211035	33.16455
1.79	3.537208	32.61726
1.8	3.857935	32.07277
1.81	4.17324	31.5305
1.82	4.483138	30.98978
1.83	4.787636	30.44981
1.84	5.086732	29.90958
1.85	5.380411	29.36789
1.86	5.668643	28.82321
1.87	5.95138	28.27365
1.88	6.228547	27.71673
1.89	6.50004	27.14925
1.9	6.765708	26.56688
1.91	7.025345	25.96368
1.92	7.278658	25.33124
1.93	7.525229	24.65715
1.94	7.764451	23.92224
1.95	7.995402	23.0951
1.96	8.216601	22.11988
1.97	8.425454	20.88534
1.98	8.616722	19.12677
1.99	8.776409	15.96868
2	#NUM!	#NUM!

with reference to the Ladder



0.5	-60.7656
0.51	-60.4235
0.52	-60.069
0.53	-59.7025
0.54	-59.3241
0.55	-58.9342
0.56	-58.5329
0.57	-58.1206
0.58	-57.6975
0.59	-57.264
0.6	-56.8203
0.61	-56.3667
0.62	-55.9035
0.63	-55.431
0.64	-54.9496
0.65	-54.4595
0.66	-53.961
0.67	-53.4545
0.68	-52.9402
0.69	-52.4185
0.7	-51.8897
0.71	-51.3542
0.72	-50.8121
0.73	-50.2639
0.74	-49.7099
0.75	-49.1502
0.76	-48.5854
0.77	-48.0156
0.78	-47.4412
0.79	-46.8625
0.8	-46.2798
0.81	-45.6933
0.82	-45.1033
0.83	-44.5102
0.84	-43.9142
0.85	-43.3156
0.86	-42.7146
0.87	-42.1116
0.88	-41.5067
0.89	-40.9003
0.9	-40.2925
0.91	-39.6837
0.92	-39.0741
0.93	-38.4638
0.94	-37.8532
0.95	-37.2424
0.96	-36.6317
0.97	-36.0213
0.98	-35.4113
0.99	-34.802

1	-34.1935
1.01	-33.5861
1.02	-32.9799
1.03	-32.3752
1.04	-31.7719
1.05	-31.1704
1.06	-30.5707
1.07	-29.9731
1.08	-29.3776
1.09	-28.7845
1.1	-28.1938
1.11	-27.6056
1.12	-27.0201
1.13	-26.4374
1.14	-25.8576
1.15	-25.2808
1.16	-24.7071
1.17	-24.1367
1.18	-23.5695
1.19	-23.0057
1.2	-22.4454
1.21	-21.8886
1.22	-21.3354
1.23	-20.7859
1.24	-20.2401
1.25	-19.6981
1.26	-19.16
1.27	-18.6258
1.28	-18.0955
1.29	-17.5692
1.3	-17.047
1.31	-16.5288
1.32	-16.0148
1.33	-15.5049
1.34	-14.9992
1.35	-14.4976
1.36	-14.0003
1.37	-13.5072
1.38	-13.0184
1.39	-12.5338
1.4	-12.0535
1.41	-11.5775
1.42	-11.1058
1.43	-10.6384
1.44	-10.1753
1.45	-9.71647
1.46	-9.26197
1.47	-8.81179
1.48	-8.3659
1.49	-7.92431

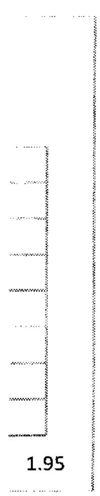
1.5	-7.48701
1.51	-7.054
1.52	-6.62525
1.53	-6.20077
1.54	-5.78053
1.55	-5.36454
1.56	-4.95277
1.57	-4.54521
1.58	-4.14184
1.59	-3.74266
1.6	-3.34763
1.61	-2.95676
1.62	-2.57002
1.63	-2.18739
1.64	-1.80885
1.65	-1.43439
1.66	-1.06399
1.67	-0.69764
1.68	-0.33531
1.69	0.023011 ← This time is the first time that matters for this project.
1.7	0.377338
1.71	0.727687
1.72	1.074072
1.73	1.416507
1.74	1.755002
1.75	2.089567
1.76	2.420211
1.77	2.746938
1.78	3.06975
1.79	3.388645
1.8	3.703618
1.81	4.014657
1.82	4.321745
1.83	4.624857
1.84	4.923957
1.85	5.218999
1.86	5.509921
1.87	5.796644
1.88	6.079062
1.89	6.357039
1.9	6.630394
1.91	6.898889
1.92	7.162199
1.93	7.419876
1.94	7.671278
1.95	7.915448
1.96	8.150868
1.97	8.374902
1.98	8.582255
1.99	8.758856

2 #NUM!

x-position of Shadow in space

y-position of Shadow in space

x		x-component velocity in space	x	f(x)
1.69	0.023011		1.69	8.657378
1.7	0.377338	35.43269	1.7	8.392406
1.71	0.727687	35.0349	1.71	8.131064
1.72	1.074072	34.63852	1.72	7.873228
1.73	1.416507	34.24344	1.73	7.61877
1.74	1.755002	33.84951	1.74	7.367555
1.75	2.089567	33.45656	1.75	7.11944
1.76	2.420211	33.06437	1.76	6.874276
1.77	2.746938	32.67268	1.77	6.631905
1.78	3.06975	32.28119	1.78	6.392156
1.79	3.388645	31.88954	1.79	6.154846
1.8	3.703618	31.4973	1.8	5.919778
1.81	4.014657	31.10393	1.81	5.686735
1.82	4.321745	30.7088	1.82	5.455479
1.83	4.624857	30.31114	1.83	5.225746
1.84	4.923957	29.91	1.84	4.997238
1.85	5.218999	29.5042	1.85	4.769619
1.86	5.509921	29.09225	1.86	4.542499
1.87	5.796644	28.67226	1.87	4.315427
1.88	6.079062	28.24181	1.88	4.087869
1.89	6.357039	27.79768	1.89	3.859182
1.9	6.630394	27.33555	1.9	3.628578
1.91	6.898889	26.84948	1.91	3.395069
1.92	7.162199	26.33101	1.92	3.157384
1.93	7.419876	25.76768	1.93	2.913836
1.94	7.671278	25.14022	1.94	2.662097
1.95	7.915448	24.41704	1.95	2.398794
1.96	8.150868	23.542	1.96	2.118701
1.97	8.374902	22.40334	1.97	1.812925
1.98	8.582255	20.73535	1.98	1.463899
1.99	8.758856	17.66008	1.99	1.025557
2	8.75	-0.88561	2	0



$$\sqrt{(x'(t))^2+(y'(t))^2}$$

y-component velcoity in space	***Speed of shadow in space
-26.4972	44.24452
-26.1342	43.70858
-25.7836	43.18123
-25.4458	42.66266
-25.1215	42.15308
-24.8115	41.65273
-24.5164	41.16192
-24.2371	40.68099
-23.9749	40.21035
-23.731	39.75049
-23.5068	39.30204
-23.3043	38.86572
-23.1256	38.44246
-22.9733	38.03339
-22.8508	37.63996
-22.762	37.264
-22.712	36.9079
-22.7072	36.57477
-22.7558	36.2688
-22.8687	35.99566
-23.0604	35.76331
-23.3509	35.58313
-23.7685	35.47201
-24.3548	35.45603
-25.1739	35.57745
-26.3304	35.90932
-28.0093	36.5889
-30.5776	37.90642
-34.9026	40.59738
-43.8342	47.25797
-102.556	102.5595

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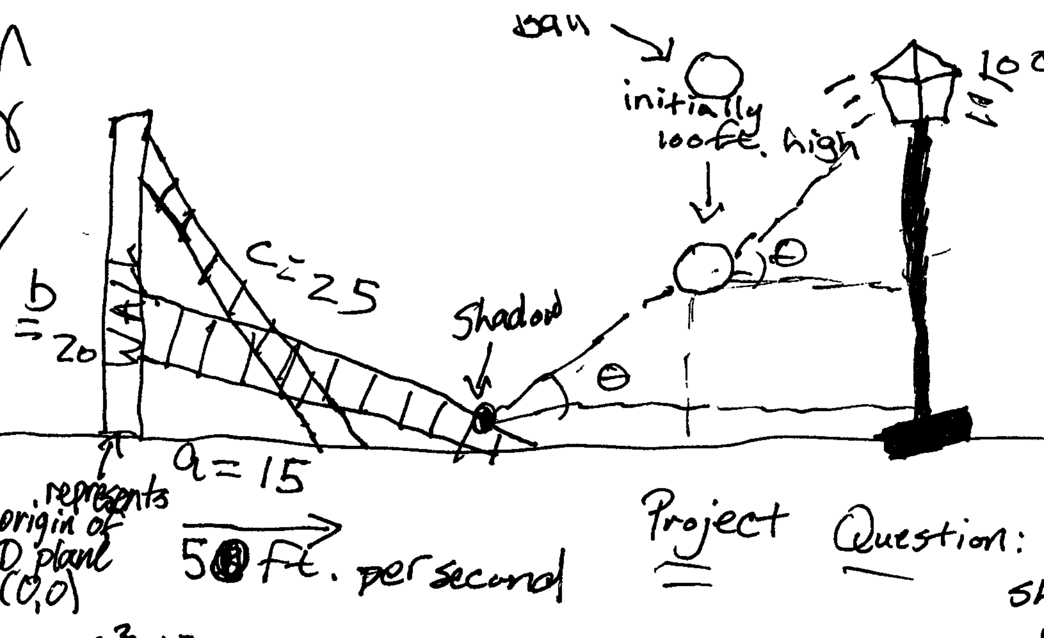

distance between ball / wall = 20 ft.
distance between ball / light = 20 ft.

Project

Question: How fast does the shadow of a falling ball move on a sliding ladder?

$a^2+b^2=c^2$

$b=\sqrt{c^2-a^2}$ $\qquad a=(15+5t)$

$b=\sqrt{625-(15+5t)^2}$

Photons emit light in every direction & resemble linear functions.

So photons travel as vectors!

Angle of shadow of ball to top of light w/respect to horizontal is equal to angle of ball to top of light w/respect to horizontal.

$a=-32\ \text{ft}/\text{sec}^2$; $V=-32t$

$\int a = v$

$\int v = p$

$V_0 =$ in... $= 0$

$P=-16t^2+P_0$

$=-16t^2+100$

$P_0 = 100$ ft

* Ladder represents linear function.

$y=mx+b$, where $m=-b/a=\dfrac{-\sqrt{625-(15+5t)^2}}{(15+5t)}$,

$b=b=\sqrt{625-(15+5t)^2}$,

* Angle from top of light to shadow of ball on ladder, w/respect to horizontal which is equal to angle from top of light to ball w/respect to horizontal at any given time is given by: $\tan^{-1}\left(\dfrac{100-\left(\sqrt{625-(15+5t)^2}-\dfrac{\sqrt{625-(15+5t)^2}}{(15+5t)}x\right)}{40-x}\right)$

$=\tan^{-1}\left(\dfrac{100-(-16t^2+100)}{20}\right)$

#2 on next pg. is fraction of ladder traveled at any time.

So, x is equal to the function the is in the box on the next page. Also, the position of the shadow can be found at any given time in the (x,y) plane!

Finally, #3 gives the length that has been traveled so far at any time or simply the position if you imagine the ladder having markings like a ruler. This function is denoted $L(t)$

* $L'(t)$ is the speed the shadow travels instantaneously at any time.

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$$100 - \sqrt{625-(15+5t)^2} + \frac{\sqrt{625-(15+5t)^2}}{(15+5t)} \quad x = 32t^2 - 0.8t^2 x$$

$$\boxed{\frac{100-\sqrt{625-(15+5t)^2} - 32t^2}{-0.8t^2 - \frac{\sqrt{625-(15+5t)^2}}{(15+5t)}} = x = x(t)}$$

#1

* Gives the horizontal distance at any time from the wall to the shadow of the ball on the ladder!!!

* This however doesn't give how much of the ladder the ball has traveled, but by using Triangle Similarity Theorem, the fraction of the length of the ladder that has been covered at any given time can be found

In other words, the fraction of the x-direction covered at any time is proportional to the length of the ladder covered at any time!

So, $\dfrac{x}{a} = \dfrac{\text{length of ladder covered at any time}}{25}$

$$= \frac{x}{\sqrt{15+5t}}$$

$$= \boxed{\frac{100-\sqrt{625-(15+5t)^2} - 32t^2}{(15+5t)\left(-0.8t^2 - \frac{\sqrt{625-(15+5t)^2}}{(15+5t)}\right)}}$$

#2

#3

$$\boxed{L(t): \frac{(25)\left(100-\sqrt{625-(15+5t)^2} - 32t^2\right)}{(15+5t)\left(-0.8t^2 - \frac{\sqrt{625-(15+5t)^2}}{(15+5t)}\right)}}$$

$L'(t)$ is instantaneous speed at any given time of the shadow of the ball ~~on the sliding ladder~~ with reference to the sliding ladder!

$$y(t) = -\frac{b}{a}\,x(t) + b$$

$$y(t) = \frac{-\sqrt{625-(15+5t)^2}}{15+5t}\big(x(t)\big) + \sqrt{625-(15+5t)^2}$$

$\boxed{* \sqrt{x'(t)^2 + y'(t)^2}}$ = Instantaneous speed of the shadow of the ball through space.