Project 4A: Bombarding a Fortification

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1 Abstract

The project goal is to find the right parameters to get the projectile that travels at 100 feet per second from the cannon to shoot over the wall that is 300 feet away and 20 feet high. To apply calculus three concepts, we intend to include how you could make the projectile land when there is some sort of wind resistance during the path of the projectile flight. How we plan to implement this is making the curve of projectile motion by parameterizing it and including a z-axis as wind can come from any direction, not strictly North, West, East, or South. This includes diagonal direction or multiple directions making the wind a vector because it has a magnitude. This would make the cannon adjustments not only, from a certain degree, up and down, but also left to right.

2 Background

Consider the problem, I would like to use a cannon with a muzzle velocity of 100 feet per second to bombard the inside of a fortification 300 feet away with walls 20 feet high.

Figure 1: The trajectory of a projectile that travels at 100 feet per second to shoot over a 20 feet high wall that is 300 feet away.

Before starting the calculations involving wind as a vector, the initial height of the cannon and the angle of the cannon must be considered as well as some other assumptions. To begin, it is assumed that the cannon must be shot to go over the wall of the fortification, meaning the distance travelled will be just over 300 ft. It is also assumed that the initial

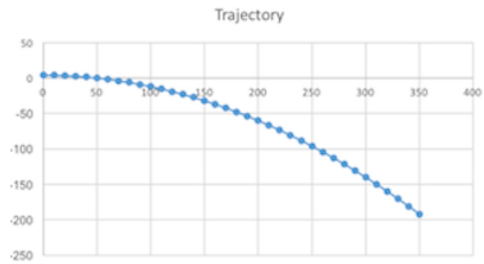

Figure 2: The trajectory of the cannon traveling $300ft$ at $100ft/s$ without adjustment to the muzzle angle or the elevation of the cannon.

height of the muzzle is approximately equal to that of a human torso, which will be $4ft$. Lastly, the ground between the cannon and the fortification is flat.

The equations used to find the horizontal component and the vertical component are $k(t) = vt + k$, v representing the horizontal velocity, t for time, and k for initial position. After substituting in known variables, the equation for horizontal projection is $k(t) = 100t$. The vertical component takes into consideration gravity and the assumption that the cannon is approximately the height of $4ft$. The equation for vertical height is $h(t) = (-gt^2/2) + vt + h$. The known values are gravity being $32ft/s^2$, initial velocity of $0ft/s$ and the assumed initial height of $4ft$. This leaves the equation for vertical height as $h(t) = -16t^2 + 4$. When using these calculations in Microsoft Excel, the trajectory of the cannon went negative after traveling only 50 ft horizontally.

Therefore, the muzzle of the cannon needs to be at an angle relative to the ground. If the height of the cannon stays constant, and only the muzzle angle (theta) changes to achieve the correct trajectory, then the new equations would be $k(t) = 100tcos(theta)$ and $h(t) = -16t^2 + 4$. With these equations, the trajectory of the cannon does not meet the desired height and distance simultaneously.

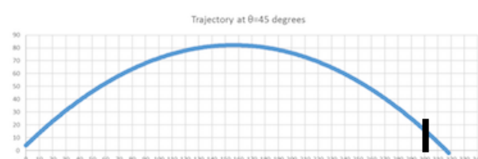

Figure 3: The trajectory after the optimal muzzle angle is applied to the equations. The black line on the graph represents the fortress.

To account for this, the cannon must have a slight elevation to fulfill the distance and height requirements. To find the optimal elevation of the cannon with an optimal muzzle angle of 46.5 degrees, the original equations can be modified once again. By trial and error with Excel and the equations, the optimal elevation of the cannon can be found by setting the equations equal to $300ft$ and $20ft$ respectfully. Starting with the vertical height equation, we can solve for t. The values of t gives the projectile incline and the maximum distance the projectile can travel. Using the maximum distance value and substituting it into the horizontal position formula, k can be solved for which will give the value for the horizontal distance of the cannon ball. With the values of t and theta, the maximum distance that can be achieved with the desired height is $296.067ft$, which indicates that the cannon would have to move forward approximately $3.9ft$.

3 Methods

With the knowledge of the optimal muzzle angle and elevation of the cannon, the wind can now be considered. Some assumptions are that wind is a vector field where all vectors are equal in magnitude, as well as the projectile has a form that of a sphere. We started by defining the projectile with arbitrary values for projectile mass and the projectile radius of $1kg$ and $0.1m$ respectfully. Next, we defined wind with arbitrary velocity and direction, which would be 8 m/s and 0.785398163 radians (this measurement is taken as the tail wind being zero and the head wind being $2pi$). The projectile moves in the $+x$ direction. Drag is a factor and the equation for drag force was derived from the previous values and assumptions. With this, we have $D = 1/2pv^2CA$, where D is drag force, p is air density (rho, this is a known value of 1.225 kg/m^3), v is the speed of the wind relative to the cannon ball, C is the drag coefficient (for a sphere, C=0.47), and A is the cross-sectional area of the cannon ball. We obtained the acceleration of the cannon ball by dividing the drag force by the arbitrarily selected mass, assuming that this acceleration is constant. After performing these calculations, we were able to use Excel to find the velocity as a Riemann sum as well as find the positional data, $z(t)$. When considering the coordinate system, the y-axis is the height of the projectile, the x-axis is the direction that the cannon is fired, and the z-axis is the crosswind. This would make the surface of the Earth on the xz-plane. Taking into account head or tail winds, we used the same technique to find these values in the x direction. Going back to the equation for horizontal distance, $k(t)$, we added the newly calculated x values and now have the equation, $k(t) =$ wind effect on distance $+(30.48 * t * cos46.5$ (for projectile velocity, v, 100 $ft/s = 30.48m/s$).

The equation for vertical height is $h(t) = (-gt^2)/2 + 30.48tsin(46.5)$. The updated plot for wind correction of the vertical height versus the horizontal distance is shown in Fig.4.

With the equation for vertical height of the projectile being $h(t) = (-gt^2)/2+30.48tsin(46.5)$, the muzzle angle has to be considered. If we consider the possible potential horizontal muzzle angles needed for correction to a straight line (in radians), then we can find the maximum value of these potential muzzle angles. In this case, the muzzle angle correction when wind is incorporated is 0.029318051 radians. With this new information, we can redefine the velocity in the z direction to take into account the muzzle angle correction for

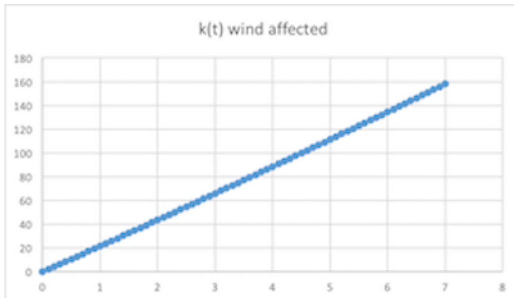

Figure 4: The horizontal distance with the effect of wind

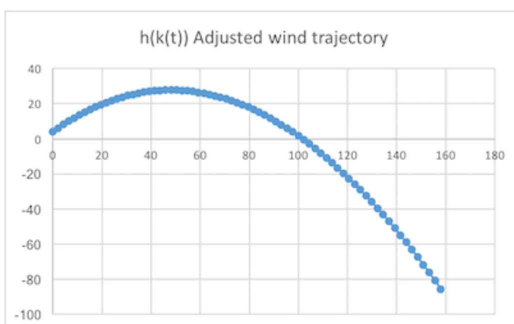

Figure 5: Vertical versus horizontal components with and adjustment for wind

wind as well as the velocity calculations. We found the Riemann sum of the new velocity to find the positional data, $z(t)$.

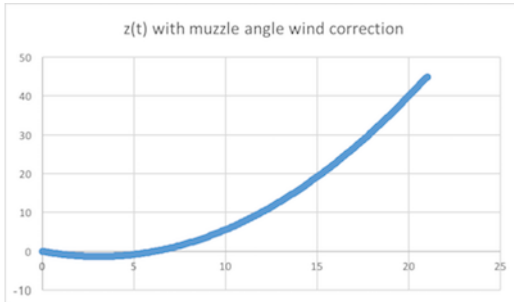

Figure 6: Positional data adjusted with muzzle angle and wind correction.

Combining all of the components and angular data that take into account the wind correction, we produced a graph that depicts an overhead view of how the trajectory would behave.

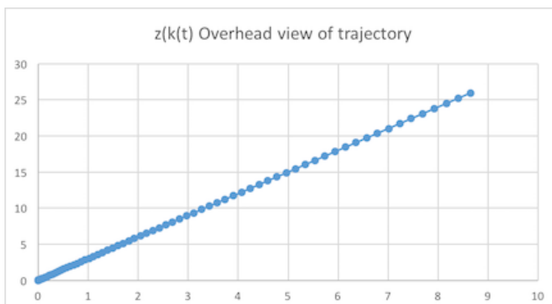

Figure 7: Overhead plot of trajectory

4 Results and Discussion

With the same concept of wind affecting the trajectory of a cannonball being fired in the air over a certain distance, there is also a force called the Coriolis Force or Coriolis Effect that can effect the trajectory of a projectile. This force is caused by the rotation of the Earth in real time, as it spins the more force is acted on the projectile in motion. The longer the object is in the air the more it is subject to trajectory deviation that is caused by this force. Also, the faster the object goes, it shortens the time needed for the projectile to reach the object desired to make contact with. For this project we tried to replicate this force by using parameters of projectile velocity, tangential velocity of the Earth, the radius of the Earth, and the displacement of the Earth in a two dimensional sense because the Earth has one axis of rotation. Using these parameters, we set the origin of the cannon in the middle and as the projectile travels ?East? it is positive and ?West? is negative. To get the initial velocity of the projectile we took the projectile velocity in meters per second and multiplied it by the cosine of the muzzle angle in radians, then added the tangential velocity of the Earth or the constant of rotation. After the projectile is fired in the air there is another force being acted upon the object which is a drag force. The drag force was computed using one half of air density (1.225) multiplied by the initial velocity squared, then multiplied by the drag coefficient of a sphere (0.47), then finally multiplied by the area of a sphere. After getting the force we calculated the drag acceleration using the drag force over time. Then, we got the velocity of the projectile considering both the Coriolis force plus the drag force by taking the change in time times the change in acceleration, all divided by two then added the previous velocity. To do this at an instantaneous time you would just take integral of the acceleration with respect to time on an arbitrary interval that you desire. Position is calculated the same way but instead of using acceleration you use the velocity. This position can now allow you to calculate the total displacement that is caused by the effect by subtraction the location from the earth displacement that is calculated by height function h(t) times the time. The results of this experimentation confirms that overtime the displacement grows and grows and will change when changing values of projectile velocity. These two parameters are both time dependent showing that the force is a parametric curve as the position is a function of time. There were some complications with the results as they seem to be rather large and do not reflect to the scale given to us in the project parameters. There is likely some error in the excel file that we cannot find or maybe excel is not strong enough to properly compute these values to give us a desired result. Nevertheless, even with skewed results it still shows a relation of the force acting upon the object and making the magnitude of the displacement larger as time goes on.

5 Conclusion

In conclusion, wind as a vector field can have a large impact on the motion of a projectile. There are several angular factors to be considered in 3 dimensions and causes the problem to become very complex. Adjusting the muzzle angle can help, but there are still angular factors for each dimensional component.

The Coriolis effect is something that needed to be investigated in further detail. With more time, research and resources, a definite answer could have been achieved. In this problem, we made several assumptions, however, in reality, wind is not constant at all points in a vector field. In fact, it is not even 2 dimensional as we have it represented. It would be interesting to explore how the projectile is affected by a non-constant 3 dimensional vector field. Also, there were some angular complexities that were greatly simplified for the sake of practicality. With this problem being in the third dimension, the changes of the muzzle angle in any direction would cause effects in all of the dimensions rather than only 2 dimensions per angle, as we have it simplified to.

6 References Cited

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