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MTH 229

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Can a Sailing Boat go in the Direction Opposite to the Direction of the Wind?



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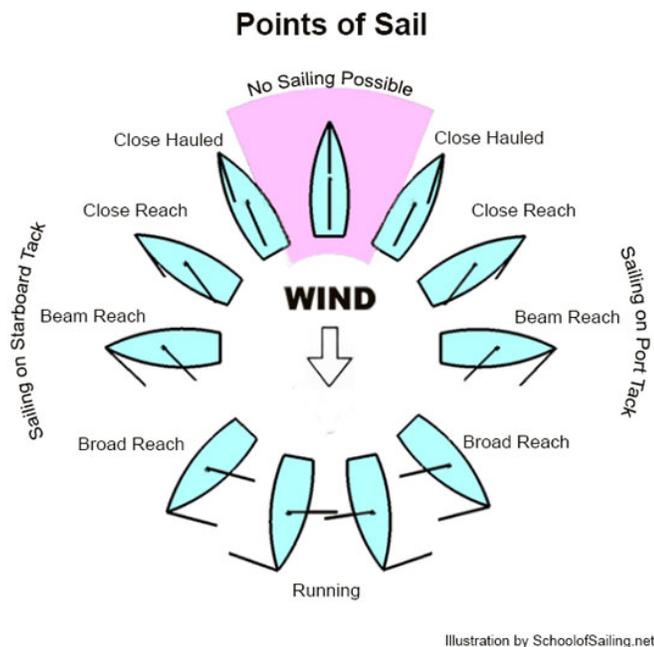
Abstract

For my final project in Calculus class, I have been assigned the action of a sailboat sailing against the flow direction of the wind. The sail of a boat works almost in the same fashion as wings do on a plane. When air flows over a wing from front to back, the wind that flows over the top of the wing travels farther than the wind that travels underneath, which creates a pressure difference, and this is what lifts the plane. For a sailboat, wind blowing at an angle against the boat inflates the sail. The sail forms a similar shape to a plane wing, which creates a difference in pressure that pushes the sail in a direction perpendicular to the wind. For the experiment, I decided to test how the sail angle can affect the sailboat's

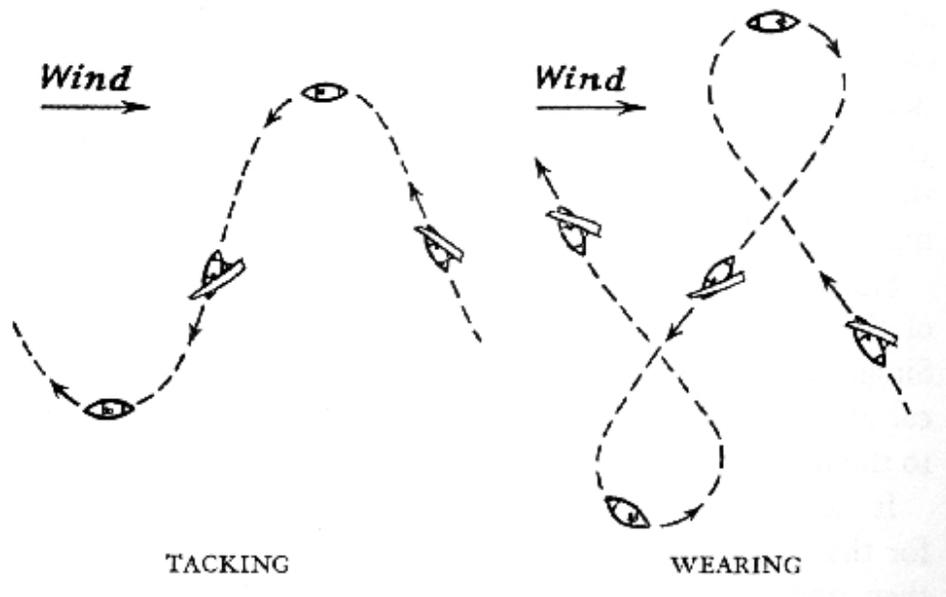
velocity. This can also show how fast a sailboat goes when it's traveling diagonally into the wind.

Can Sailboats Sail Into the Wind?

It's obvious that a sailboat can't sail directly against the wind, as it would just be pushed backwards with the direction of the wind, as demonstrated here:



So how do sailboats get where they need to go if it's opposite the direction of the wind? They slice through the wind at different angles, veering back and forth in a zigzag motion (tacking), or they make a loop and cut back the other direction diagonally against the wind (wearing), as shown in the diagram below:

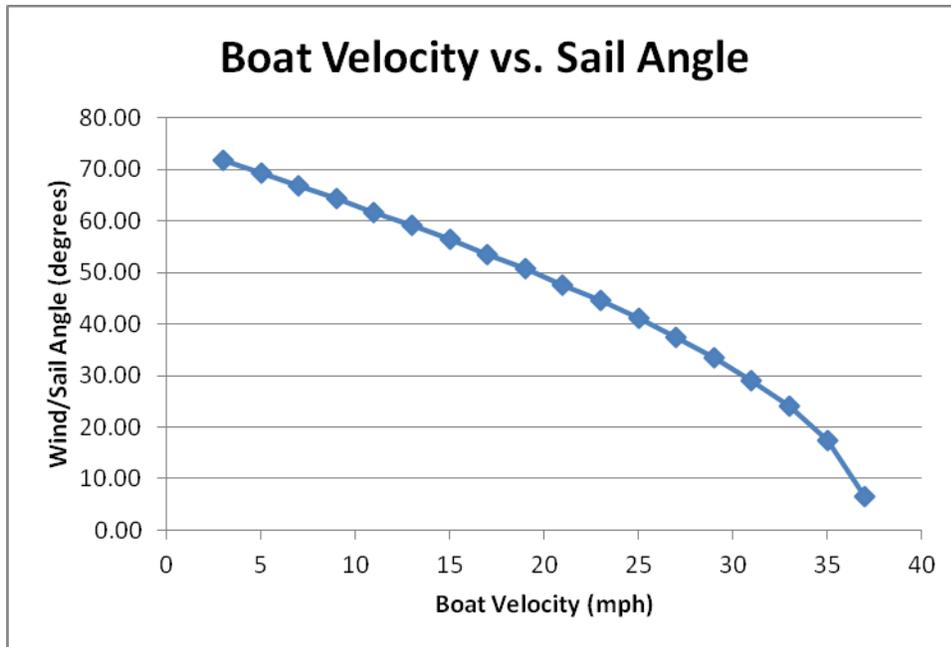


Experimentation and Variables

At first, I tried using an equation that related the wind force and the drag force on the sailboat to the boat mass and the acceleration, one that related the drag force, water pressure, and keel area to the air pressure and exposed boat area, one that related the wind force to the wind pressure and the sail area, Edmund Bernoulli's equation ($P + \frac{1}{2}\rho V^2 + \rho gh = C$), and Euler's equation ($\frac{dP}{dn} = \rho V^2/R$). I came to realize quickly that these equations had too many variables to solve for and were really just unnecessary. Instead, I decided to use an equation that allowed me to manipulate the velocity of the boat, which produced differing wind (or sail) angles in order to match these velocities.

Equations and Calculations

I decided to use the equations $A = \sqrt{W^2 + V^2 + 2WV \cos \alpha}$ and $\beta = \arccos \left(\frac{W \cos \alpha + V}{A} \right)$. In this equation, A stands for the apparent wind velocity (in mph), W stands for the true wind velocity (mph), V stands for the velocity of the boat (in mph), α is the true pointing angle (in degrees), and β is the wind (or sail) angle (in degrees). I decided to make the apparent wind velocity constant at 50 mph, the true wind velocity constant at 30 mph, and the true pointing angle constant at 65° so I could manipulate the velocity of the boat, which would in turn produce different results for the angle of the sail.



Conclusion

In conclusion, I found that by manipulating the two equations I had, I could show how a change in boat velocity would result in a change in the sail angle, and vice versa. This was about as mathematical of an approach as I could have taken without going in way over my head and having too many variables to keep up with. This project had not been done

before me, so there was nothing I could compare to or old tests I could try to re-run myself, and I was left to come up with findings on my own. However, I was able to display a logical conclusion, which is basically all the project was asking for.

References

https://en.wikipedia.org/wiki/Apparent_wind

<https://www.nationalgeographic.com/volvoceanrace/interactives/sailing/index.html>