# Evaluating the Ability of Different Blades

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**Abstract**

In this paper, we will use the already-determined, optimal motion of a blade for cutting ability, to compare separate blade types with regard to their affinity for cutting through a medium. The cutting ability of different blades will be measured and determined by using variables relevant to blade shape. This also includes handling motion and orientation, and as well as the velocity of the cutting blade. By defining these variables for a given blade, a model will be constructed to compare multiple different blades, and a relative measurement for cutting ability will then be implemented and used to decide which of those blades is optimal.

## 1 Introduction

Assume a situation where a solid block of material is to be cut into two pieces by a blade swinging through it. To cut through the material, a blade must travel through a cross-section of this material to separate it. The area of the cross-section that is cut is entirely dependent upon the type of blade that is used. Therefore, we can observe how different blades can either maximize or minimize this area, resulting in the question of what blade aspects contribute to this cutting behavior.

When a blade is cutting through a medium, it has a cutting ability that is dependent upon the shape of the blade and the motion it takes. For this project, we have specific assumptions in order to prepare a model for evaluating this cutting ability. By beginning with previous research done on the optimal motion of a blade for cutting through a medium, we assume that the most optimal motion a blade can take while cutting is one where the cut is penetrating, or propagating, at a uniform rate. Furthermore, this type of cutting was observed with specific blade shapes from this earlier research.

Another main assumption for this project is the idea that the propagation of a cut from a blade is created from a rotational motion of the blade swinging around in a circular motion. This is possible from either a blade rotating on a fixed axis, or from a fixed arm holding the blade swung around shoulder. For this project, the motion of a blade will be assumed to rotate around an arbitrary circle that mimics these types of blade propagation.

With these beginning assumptions, we can qualitatively model the path of a specific type and shape of blade as it cuts through a medium. The evaluation of the cutting ability between different blades is applicable to many fields, as this analysis can be used in a variety of settings to increase efficiency of physical tasks. Through an accurate model, this defined effectiveness of an arbitrary blade as it passes through a material can be observed. Using all of this allows for the completion of the goal of this project, which is determining a method to decide which blade is more effective when compared to other given blades.

In this project, we begin with an examination of the progression of a blade through a material with regard to its shape and rotated motion. Then, measurements of the behavior of this propagation are defined in order to compare different blades. Finally, a set of required measurements of the observed blades can be taken to compare and evaluate their cutting ability. These measurements are organized into a final model, which is simply a method of comparing any given blade to another, in order to determine the relative effectiveness of any given blade.

### 1.1 Previous research

Research performed on the optimal blade shape for cutting is used as a foundation for this project. In this research, various representations of sword shapes were constructed using parametric equations, and their rotations were graphed to represent a swinging motion of the blade. From this process, it was determined that the rotated curves had three possible behaviors. The rotations either intersect their original curves, continually separate from their original curve, or remain the same distance apart from their original curve at every point ([Sav17]).

In Figure 1, four examples of these possibilities are represented. In **A** and **B**, the rotations have a growing separation from their initial curve. In **C**, the rotations intersect back onto the initial curve. In **D**, the rotations are a uniform distance away from the initial curve at every point. From

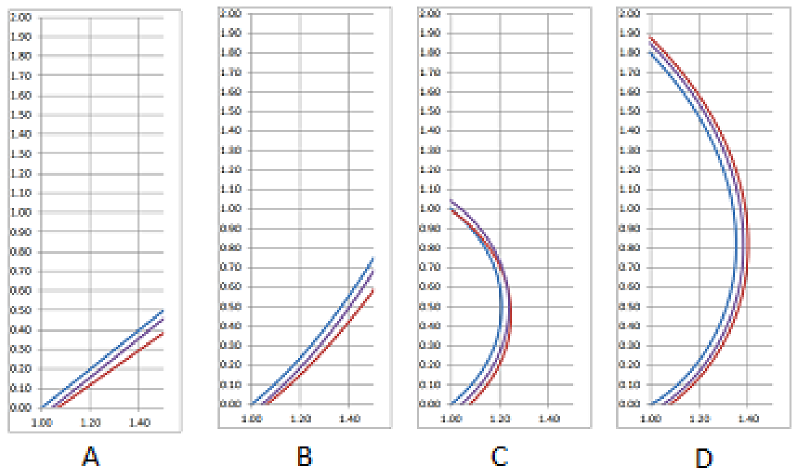


Figure 1: Examples of curves rotated from an initial parametric curve

the conclusion of this previous research, the case seen in **D** was decided to be most efficient and effective with regard to the cutting of a blade curve. Therefore, the idea of this type of propagation of a blade creates a motivation for comparison of propagating behavior between different blade shapes, and their relative cutting effectiveness.

### 1.2 Method of cutting

By beginning with a rudimentary definition of cutting, parameters of modeling of a cutting edge can be defined. Our informal definition of cutting with a blade is described as the action of rotating a blade around while applying a pulling motion in one direction, parallel to a medium with a cutting-edge. Therefore, a main assumption of this project is that the modeled blades will have to achieve this description of cutting, as opposed to only a penetrating force where a blade is only being pushed into a medium and has no motion parallel to the surface of the medium.

### 1.3 Cutting model of uniform propagation

To begin with the construction of the model, this previously defined concept of cutting is applied by attempting to model the edge of a blade as a parametric curve in a Cartesian plane. A medium's resistance to a blade

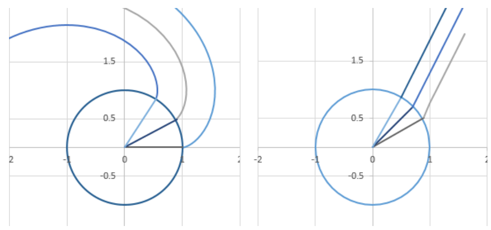


Figure 2: Uniform propagation of an involute of a circle and a straight-edge

is dependent upon the speed of the propagation of the blade. Therefore, a curve that is propagated uniformly at each point as it is moving through a medium will provide the most consistent and therefore, effective cutting motion ([MMS60]).

Uniform propagation is defined as a change in position of a curve or line, where the distance of separation between each point on the original to the propagated, or rotated, version is constant. However, only a straight line and a spiral can be uniformly propagated through rotations without changing size or shape. We can observe both the involute of a circle and also a line for an example of how these shapes can be uniformly propagated.

**Definition 1** ([BW17])**.** The involute of a given circle is a curve mapped by

$$x = a\cos\phi + a\phi\sin\phi,$$
$$y = a\sin\phi - a\phi\cos\phi,$$

for $x, y \in \mathbb{R}$, where $a$ is the radius of the given circle, and where $\phi$ is the angle of the $x$-axis to any radius of the given circle.

The ability of both the involute and an arbitrary straight line to be uniformly propagated can be seen with subsequent constructions.

In Figure 2, an arbitrary circle is drawn with radii at three different angles to represent a swinging motion at a length of the radius as time progresses. In this figure, the involute curves and the straight lines projecting from the edge of the circle are the modeled shapes of a blade that is held in a hand at different rotated positions. The angle between the radius and these projected lines is changing at each point to maintain uniform propagation. This changing angle is represented as the angle between the wrist and the blade in a physical hand. Therefore, to maintain this uniform propagation of

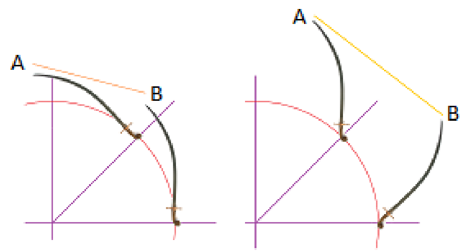


Figure 3: The distance the tip of a sword travels compared to the distance the handle travels

a blade with one of these shapes, which was assumed to be the optimal motion of a cutting blade, the user's wrist must be changing to accommodate for the changing angle.

### 1.4 Cutting model of non-uniform propagation

If the shape of the blade is any other curve, then there is no way to uniformly propagate the cut through rotations. Therefore, we must define a different method of evaluating other curves with constant and varying wrist angles. The velocity of different parts of the blade must be measured, as they will not be consistent with nonuniform propagation. An example of how the velocity of certain parts of the blade are not constant throughout the same blade are shown in Figure 3.

From Figure 3, a line is drawn from the tip of each blade as it travels from one position to the other, while maintaining a constant wrist angle. The distance that the tip has to travel is greater than the handle has to travel in the same amount of time. Therefore, the tip of the blade has a greater velocity than the handle in this case, providing another measurable variable that will be considered in the complete model.

## 2 Handling orientation

In this section, we take the initial model, and attempt to optimize the effectiveness of any blade through varying the different forms of handling. By using our initial qualitative model, numerical qualities for the path of a cutting blade as it relates to handling orientation can be derived.

## 2.1 Wrist orientation

As seen in Figure 2, the varying wrist positions are necessary for uniform propagation of the blade edge. Therefore, the variability of the angle between the wrist and the blade can be observed. Subsequently, the magnitude of this variability determines the efficiency of the blade. A blade shape requiring a greater range of different wrist orientations for optimal motion would be more difficult to handle effectively than one with a smaller range. Therefore, it is clear from Figure 2 that the involute-shaped blade is more preferable than the straight-edge blade as the range of angle variation is smaller between the involute curves than the straight lines. From this, we have one method of evaluating the cutting ability between different blades.

## 2.2 Nonuniform propagation handling

Because only a straight line and an involute can have uniform propagations without changing shape, another method of evaluating blade shapes that differ from these two must be defined. The amount that the wrist angle changes is dependent upon the amount of curvature that the curve has. To find the curvature at specific points in the model, we must use the theorem of curvature of a parametric curve:

**Theorem 1** ([RA15])**.** If $r(t)$ is a regular parametrization, then the curvature at $r(t)$ is

$$\kappa(t) = \frac{||r'(t) \times r''(t)||}{||r'(t)||^3}.$$

*Proof.* Let $v(t)$ be a parametric curve such that $v(t) = ||r'(t)||$. We know that a unit tangent vector of a parametrization is defined as

$$T(t) = \frac{r'(t)}{||r'(t)||},$$

and therefore $r'(t) = v(t)T(t)$. By using the Product Rule we get

$$r''(t) = v'(t)T(t) + v(t)T'(t),$$

and then find that

$$r'(t) \times r''(t) = v(t)T(t) \times (v'(t)T(t) + v(t)T'(t)) = v(t)^2 T(t) \times T'(t),$$

because $T(t) \times T(t) = 0$ from the definition of unit tangent vectors. It follows that because $T(t)$ and $T'(t)$ are orthoganal, we have

$$||T(t) \times T'(t)|| = ||T'(t)||.$$

So

$$||r'(t) \times r''(t)|| = v(t)^2 ||T'(t)|| = v(t)^3 \kappa(t) = ||r'(t)||^3 \kappa(t)$$

from the definition of curvature. Therefore,

$$||r'(t) \times r''(t)|| = ||r'(t)||^3 \kappa(t) \Rightarrow \kappa(t) = \frac{||r'(t) \times r''(t)||}{||r'(t)||^3}.$$

□

This theorem can be used for a parametric curve to determine the value of curvature at that distinct point in the graph. A curve with a smaller value of curvature toward the tip of the blade and a greater value toward the handle will require a larger change of the wrist angle needed to keep optimal propagation of the blade. This was seen in Figure 2, as the involute had no change in the wrist angle unlike the straight line. The straight line will have a curvature of 0 at every point, and the involute will have some decreasing value outwards from the origin due to its spiral behavior. To verify this for a line, we assume an arbitrary equation:

$$r(t) = mt + b$$

for $t \in \mathbb{R}$, where $m$ and $b$ are constants. Next,

$$r'(t) = m, \quad r''(t) = 0.$$

So by definition, there is no curvature for a straight line. For any curve that is not a straight line, there will be some value for curvature at certain points in the graph. Determining curvature for any other type of curve will provide another factor that must be considered for the cutting ability of a blade.

## 2.3 Handling with varying velocity

As explained in Section 1.4, a part of a blade will move at different velocities if the wrist angle is constant and the shape of the blade is not a line or an involute of a circle. This behavior can either improve or hinder a blade's cutting ability. For this section, rotations of a curve are used to mimic a

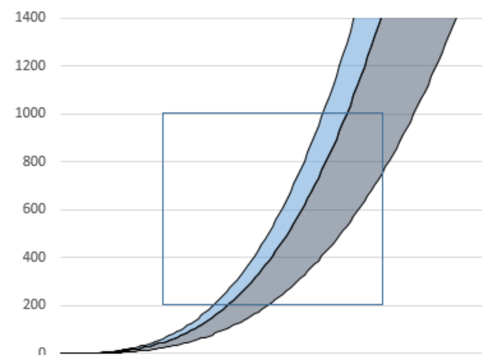


Figure 4: The area bounded by three rotations of $f(x) = x^3$ in a certain square area.

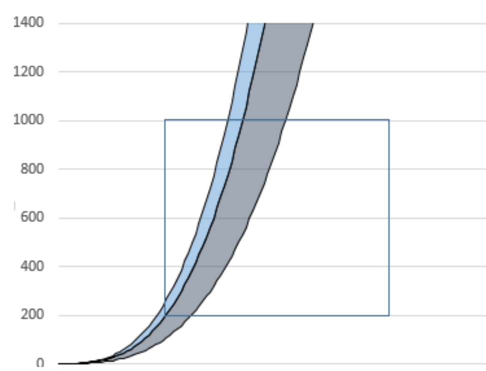


Figure 5: The area bounded by three rotations of $f(x) = 4x^3$ in a certain square area.

constant wrist-angle, and therefore each rotation is a result of this specific handling orientation. To give an example of this, we can use two examples of different shaped curves rotated at different instances.

In Figure 4 we have three rotations of the same curve, $f(x) = x^3$. A square is plotted at a specific position to observe the change in area between the two bounded regions at that specific point. It also follows that the area that is bounded by these rotations represents the area of material that would be cut with a blade of this shape cutting through.

In Figure 5 we have a similar representation, with the curve of $f(x) = 4x^3$ instead. There is a square equivalent to the first in Figure 4, and also plotted at the same location to accurately observe the difference in area between both curves.

After qualitatively analyzing the difference between the area covered between the curve at each instant, the propagating behavior of a blade with parts that have varying velocities can be determined.

We can use vector calculus to quantitatively measure the pattern of propagation from a specific blade. To complete this, we want to first construct normal vectors at two distinct points on the given curve. These points represent any two unique points on the blade which will engage in cutting. Next, a given rotation of the original curve is plotted to represent a cutting motion moving through space. Finally, the distance between each normal and its intersection of the rotated curve is determined. Finding the absolute value of the difference of these distances then gives a measurement for propagation behavior.

We begin with a continuous function $f(x)$. For the tail of the normal vector of $f(x)$ we have the coordinates $(x_n, y_n)$, and for the coordinates of the intersection of this vector with a rotated curve we have $(x_i, y_i)$. The distance from $(x_n, y_n)$ to $(x_i, y_i)$ for two unique points on the curve of $f(x)$ is found, and the absolute value of difference of their distance, $\delta$, is determined,

$$\delta_{1,2} = \left| \sqrt{(x_{1_i} - x_{1_n})^2 + (y_{1_i} - y_{1_n})^2} - \sqrt{(x_{2_i} - x_{2_n})^2 + (y_{2_i} - y_{2_n})^2} \right|.$$

Here, $(x_1, y_1)$ and $(x_2, y_2)$ are used to represent the two given points on the function, or curve of the blade. By using all of these variables, we can form an example of this representation. In Figure 6, we have a graphical representation of what this measurement represents for the function $f(x) = x^2$. A rotation of this curve approximately $-14°$ is also plotted, with two normal vectors intersecting at two distinct points. The value for $\delta$ for this

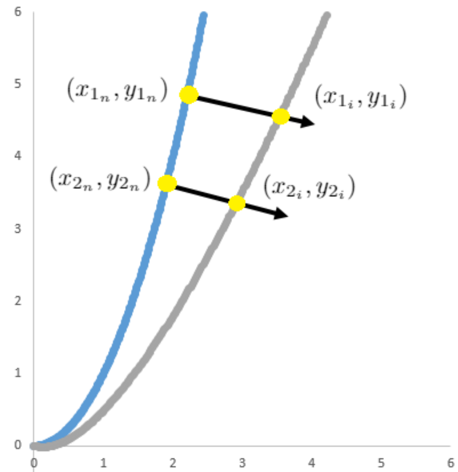


Figure 6: A function representing blade shape, $f(x) = x^2$ (blue), a rotation of that function (grey), and the normal vector of $f(x)$ intersecting the rotation from two points $(x_{1_n}, y_{1_n})$ and $(x_{2_n}, y_{2_n})$

function can then be determined when given initial values for $(x_1, y_1)$ and $(x_2, y_2)$.

Therefore, $\delta_{A,B}$ for two different positions on a function, $A = (x_1, y_1)$ and $B = (x_2, y_2)$ such that $A \neq B$, determines propagation behavior of that function when rotated. It follows that $\delta = 0$ implies that the distance is constant for every position $A$ and $B$ on a blade, or that the function is uniformly propagated. A blade curve with a greater value of $\delta$ at every position will give less uniform propagation, and that blade will therefore have a less effective cutting-ability.

Further analysis of this measurement of propagation of a rotated function can be used to give a more accurate measurement when the wrist-angle is not constant. By using this idea of the value of $\delta$, an attempt to minimize the value of $\delta$ through changing the wrist-angle while cutting can be researched.

## 3 Realistic variables & constraints

In this section, we work with realistic variables of a blade to further develop the accuracy of the comparison model. Then, a more applicable method of evaluation of the cutting ability of a blade can be constructed and used in

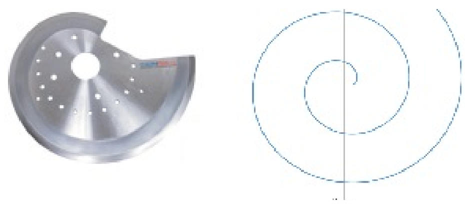

Figure 7: An involute-shaped slicer blade with an involute curve for comparison. Photograph from the G.W. Steffens corporation, http://www.gwsteffens.com/slicerblades.html.

final model.

### 3.1 Realistic constraints

When constructing a model of a physical blade cutting through a medium, realistic limitations must be defined. When swinging a blade through a medium, there is only a specific range of motion that is possible. Therefore, every variable that is considered when evaluating a blade will be measured over a general range, with limits that are defined by what is physically possible when handling a blade. The possible angles between the wrist and the blade in a hand are between $0°$ and $90°$ on a unit circle. Also, we find that the propagation behavior has a specific behavior for one quadrant of a circle, and then repeats the same behavior in a mirrored-fashion around the rest of the circle. As a result, the curves modeling the path of a blade only need to be considered on that specific interval. Therefore, for any motion outside of this interval, the modeled path of a blade will behave similarly if the motion is around an unchanging circle that represents the fixed method of rotation.

### 3.2 Realistic blade shapes

In order to allow this model to become applicable, realistic blade shapes must be considered. As seen in previous sections, the straight-edge blade was the most trivial example as it can be easily constructed without assumptions. However, other blade shapes are more specific and require more assumptions when representing as a function.

To provide an example of depicting a blade shape as a parametric curve, we can use a common blade for cutting cheese and meats, which is the previously shown involute.

In Figure 7, we have a constructed involute, and a realistic blade with the exact same curve as its cutting edge. Commercial blades of this shaped are used because of the proposed assumption that the uniform propagation of the blade through a medium is most effective and efficient. This blade shape is rotated around a fixed axis, fulfilling the beginning assumption of this project with regard to blade motion. Therefore, we have a real example of a blade shape where the developed model of evaluation can be applied. Clearly, the propagation is uniform as it is rotated around its axis, so it will have optimal motion as it progresses through a medium. Next, we can see that there is no need for a "wrist-angle" change with this shape because its shape has an increasing curvature towards the axis, and also because it is not being held in a physical hand, so no human limitations are a factor. Therefore, from this brief analysis we can find real measurements for the shape of a realistic blade that can be used to construct a model for evaluating its relative effectiveness. By using this as an example, we can see that this form of evaluation can be performed with many more blade shapes that can be represented as a parametric curve.

# 4 Comparison of distinct blades

In this section, we compare distinct types of swords and blades using the established method of measuring a blade's cutting ability with the limiting factor of efficiency. Using the model, a qualitative and quantitative set of data is obtained for each blade being examined. A preferred choice between multiple different blades can then determined because of explained reasons regarding the nature of their shape and motion.

## 4.1 Blade evaluation model

In this section, the criteria for the complete model for determine the relative cutting-ability of a blade is described. The completed model assesses three specific properties for any two blades as a method of comparing effectiveness of the cut. The following properties can be used for this and are listed in order of importance:

1. Propagating behavior (uniform vs. nonuniform)
2. Variation of propagated area ($\delta$)
3. Wrist-angle variation

First, propagating behavior of each blade is analyzed, and the potential for them to be uniform or nonuniform is decided. If one blade can uniformly propagate and the other blade cannot, then the uniformly-propagating blade will provide a more effective cutting-ability than the other.

Next, the variation of propagated area is determined through finding the value of $\delta$ for each blade, where position $A$ is the part of the blade closest to the handle, and the position $B$ is the tip of the blade. Therefore, $\delta_{A,B}$ will give the difference between the normal vector at the two bounds (handle and tip) of each blade. A minimization of this value implies propagation behavior that is most similar to uniform, and also implies that a minimization of this yields a more effective cutting-ability.

Finally, the required variation for the handling orientation of the wrist-angle is measured. By graphing rotations of the curve of the blade at specific intervals, the range of the varied angle between the wrist and the blade is determined. This range of possible wrist-angles is compared with another given blade, and the blade with the smaller range will yield the more efficient and useful blade. The other method of determining handling requirement of a blade shape is performed by quantitatively measuring the curvature, $\kappa$, approaching the handle of the blade. A smaller value of $\kappa$ will require a larger change in the wrist angle, and therefore is less effective than a blade with a larger value of $\kappa$ approaching the handle.

By measuring a specific blade with these parameters, its effectiveness will be determined. Then a direct comparison between different blades can be made in order to evaluate which blade is most effective for the given conditions.

## References


[BW17] Frederick H. Bailey and Frederick S. Woods. *Analytic Geometry and Calculus.* The Athenaeum Press, Boston, Massachusetts, 1917.

[MMS60] Marey-Monge and Guillaume Staislaus. *Memoir on Swords, translated from the French by Henry Hamilton Maxwell.* John Weale, London, 1860.

[RA15] John Rogawski and Colin Adams. *Calculus 3rd ed.* W. H. Freeman, 41 Madison Ave., New York, NY, 2015.

[Sav17] Peter Saveliev. Discrete calculus. Unpublished manuscript, 2017.