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Math 229
4 December 2016

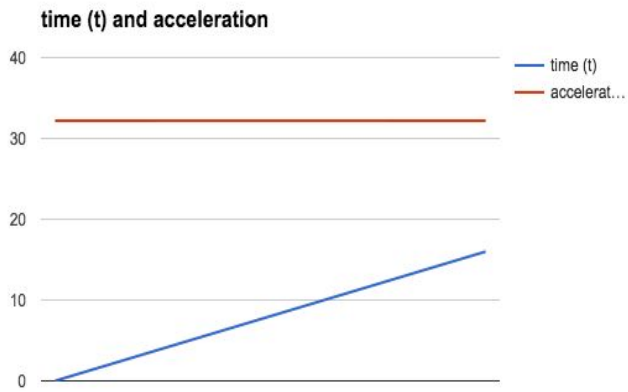
The Proper Way to drop a ball off of a building

Scenario: I was asked to find a way to throw a ball off of a one hundred story building so that it would hit the ground at one hundred feet per second. In a real world scenario this would act as if someone had asked you to go to the top of a skyscraper and drop some circular object off the top to finish at a certain speed and show why or why not it would work.

Process: I first started by trying to find the height of one story. I found that a story can range anywhere from 12 feet to 15 feet. I chose to use 13 feet for my work. I multiplied by 100 to get 1300 feet for the height of the building. I followed by remembering that if I started with an acceleration graph being constant I could use antiderivatives to find an equation for velocity and height. I started with acceleration equation of **(dy')'=a** then took the antiderivative to get **(dy')=at** I took one more antiderivative to get the height **dy=½*at^2**

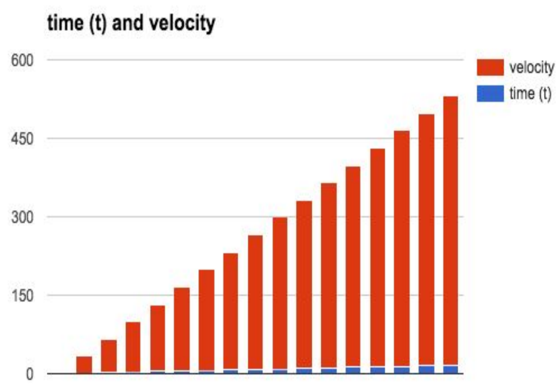
time (t)	acceleration
0	32.2
1	32.2
2	32.2
3	32.2
4	32.2
5	32.2
6	32.2
7	32.2
8	32.2
9	32.2
10	32.2

11	32.2
12	32.2
13	32.2
14	32.2
15	32.2
16	32.2



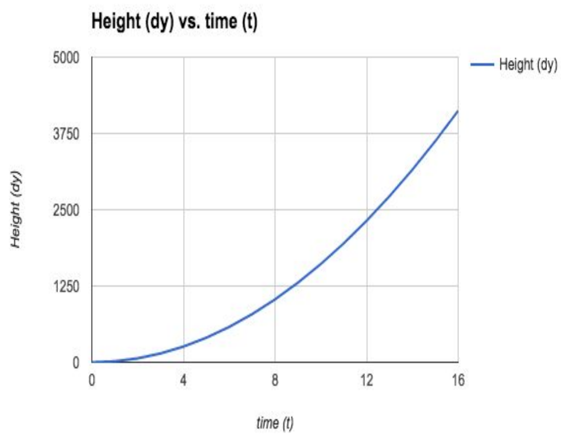
time (t)	velocity
0	0
1	32.2
2	64.4
3	96.6
4	128.8
5	161
6	193.2
7	225.4
8	257.6
9	289.8
10	322

11	354.2
12	386.4
13	418.6
14	450.8
15	483
16	515.2



time (t)	Height (dy)
0	0
1	16.1
2	64.4
3	144.9
4	257.6
5	402.5
6	579.6
7	788.9
8	1030.4
9	1304.1
10	1610

11	1948.1
12	2318.4
13	2720.9
14	3155.6
15	3622.5
16	4121.6



I then started more analytically using the formula **dy=½*at^2** where **dy** was the height, **a** was 32.2 feet per second aka the acceleration due to gravity, and **t** was time. I wanted to get an estimate of how long it would take a ball to fall until it hit the ground.

dy=½*at^2

1300ft=½*32.2ft/sec*t^2

t^2=80.75

t=sqrt(80.75)=8.98sec

It takes approximately 9 seconds for the ball to hit the ground. I took this information to move to the next step of trying to find the velocity for dropping a ball off the top of the building so that it finished at 100 feet per second. For this I used the equation $\mathbf{V_f=V_o+at}$ where $\mathbf{V_f}$ was the final velocity, $\mathbf{V_o}$ was initial velocity, and **at** was still acceleration due to gravity and time.

$\mathbf{V_f=V_o+at}$

$\mathbf{100ft/sec=V_o+(32.2)*(9)}$

$\mathbf{V_o=-189.8ft/sec}$

This result told me that for a ball to be dropped straight down I had to have an initial velocity of -190ft/sec which is impossible.

time (t)	velocity	initial velocity to get 100ftpsec
1	32.2	67.8
2	64.4	35.6
3	96.6	3.4
4	128.8	-28.8
5	161	-61
6	193.2	-93.2
7	225.4	-125.4
8	257.6	-157.6
9	289.8	-189.8

This chart comes from excel where I used my results of finding time taking 9 seconds followed by velocity only increasing due to gravity each second to show that at 9 seconds it needs -190ft/sec to reach 100ft/sec at impact.

From here I started to think that the negative initial velocity was telling me to start by throwing the ball up first and let it fall from there.

V_f=sqrt((2*32.2*dy)+V_i^2)

I used this equation to show the ball traveling upwards

Vi	Height (dy)	Vf
0	1300	289.3440858
0	1350	294.855897
0	1400	300.2665483
0	1450	305.581413
0	1500	310.8054054
0	1550	315.9430328

0	1600	320.9984424
0	1650	325.9754592
0	1700	330.8776209
0	1750	335.7082066
0	1800	340.470263
0	1850	345.1666264

This idea did not prove to work because excel showed that increasing the initial height that the ball was thrown from ended up increasing the final velocity at impact instead of decreasing like I wanted.

After this testing I realized, that there was not a way the drop a ball or throw it upwards that would result in the ball making contact with the ground at 100ft/sec. I went on to thinking what height would work to drop a ball from to let it make impact at 100ft/sec. I used the same equation as I did for increased height, $\mathbf{V_f=sqrt((2*32.2*dy)+V_i\string^2)}$, but I lowered dy instead of increasing it.

Vi	height	Vf
0	1300	289.3440858
0	1250	283.7252192
0	1200	277.9928057
0	1150	272.13967
0	1100	266.1578479
0	1050	260.0384587
0	1000	253.7715508
0	950	247.3459116
0	900	240.7488318
0	850	233.9658095
0	800	226.9801753

0	750	219.7726098
0	700	212.3205124
0	650	204.5971652
0	600	196.570598
0	550	188.2020191
0	500	179.4435844
0	450	170.2351315
0	400	160.4992212
0	350	150.1332741
0	300	138.9964028
0	250	126.8857754
0	200	113.4900877
0	150	98.285299
0	100	80.24961059
0	50	56.74504384
0	0	0

I saw that it would be around 150 feet high for the ball to attain a final velocity of close to 100ft/sec so I used a height close to that to find my answer.

Vi	height	Vf
0	150	98.285299
0	151	98.61237245
0	152	98.93836465
0	153	99.26328626
0	154	99.58714777
0	155	99.90995946
0	156	100.2317315

My findings showed me that dropping a ball from 156 feet height with an initial velocity of 0 would result in the ball making contact with the ground very close to 100 feet per second. After this I wanted to know if I changed initial velocity like someone had thrown the ball instead of dropping it would it change the height needed much.

Vi	height	Vf
15	1300	289.7326354
15	1250	284.1214529
15	1200	278.3971983
15	1150	272.5527472
15	1100	266.5801943
15	1050	260.4707277
15	1000	254.2144764
15	950	247.8003228
15	900	241.2156711
15	850	234.4461559
15	800	227.4752734
15	750	220.2839077
15	700	212.8497122
15	650	205.1462893
15	600	197.1420807
15	550	188.7988347
15	500	180.0694311
15	450	170.8947044
15	400	161.1986352
15	350	150.8807476
15	300	139.8034334
15	250	127.7693234

15	200	114.4770719
15	150	99.4233373
15	100	81.63945125
15	50	58.69412236
15	0	15

Again I found even with an initial velocity of 15ft/sec that the height needed would be close to 150 feet.

Vi	height	Vf
15	150	99.4233373
15	152	100.0689762

Looking closer, if the initial velocity of the ball was 15ft/sec then the height needed to end at 100ft/sec would need to be 152ft.

I followed these findings up with the question of how long would it take the ball to hit the ground then from the height that was successful. I went back to my equation **dy=½*at^2** and plugged in 156ft for when the initial velocity was 0ft/sec and 152ft for when initial velocity was 15ft/sec

dy=½*at^2

156ft=½*32.2ft/sec*t^2

t^2=9.69

t=3.11 seconds

dy=½*at^2

152ft=½*32.2ft/sec*t^2

t^2=9.44

t=3.07 seconds

In conclusion, this project showed me that there is no way to throw a ball off of a one hundred story building and have it hit the ground at 100 feet per second. To do so required drastically lowering the height that the ball was thrown from to achieve this goal.

Resources: Parents, neighbors,
http://www.physicsclassroom.com/class/1DKin/Lesson-6/Kinematic-Equations-and-Free-Fall